July 30, 2019

ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa

final exam
Time allotted: 3 hours
Professor: Louis Hotte

NB This questionnaire has 3 pages.

1. (20 points) Consumer preference relations
Let vector $\vec{x}$ denote a consumption bundle, with $\vec{x} \in X = \Re^n_+$. Let $\succsim$ denote the binary preference relations between any two bundles such that $\vec{x}^1 \succsim \vec{x}^2$ implies that bundle 1 is at least as good as bundle 2.

a) Which property of the preference relations implies a diminishing marginal rate of substitution (MRS) between two goods? Explain with the help of a graph. Make sure to explain properly what diminishing MRS means.

2. (40 points) Consumer theory
Suppose that a consumer's welfare depends on the quantities of agricultural goods x_1 and manufactured goods x_2 that she consumes. Suppose more precisely that her utility level can be represented by the following utility function:

$$u(x_1, x_2) = (x_1 - \alpha_1)^{1-\theta}(x_2 - \alpha_2)^{\theta},$$

where α_i are positive parameter values and $\theta \in (0, 1)$. The respective prices of the goods are p_1 and p_2. The consumer's income is y.

a) (5) Write down the consumer's problem.

b) (10) Express the Langragian function for this problem and give the first-order conditions.

c) (15) Derive the *indirect utility function*. (Explain briefly your steps. If you don't, I can't give much partial marks in case you make algebraic mistakes.)

d) (5) Derive the (ordinary) demand functions.

e) (5) If you were provided with the indirect utility function only, how would proceed to find this consumer's demand for manufactured goods?

3. (40 points) The Nash equilibrium in a soccer penalty kick[1]

Consider the penalty kick in soccer. There are two players, the goalie and the striker. The striker has three strategies: kick to the goalie's right (R), to the goalie's left (L) or to the center (C). The goalie has three strategies: move left (L), move right (R) or stay in the center (C). Let α be the probability that the kick is stopped when both choose L and let β be the probability that the kick is stopped when both choose R. Assume that $0 < \alpha < \beta < 1$. Consequently, the striker is more skilled at kicking to the goalie's left. If both choose C, the goalie stops the ball with certainty. The payoff matrix is as follows.

		Striker L	Striker C	Striker R
Goalie	L	α, $1-\alpha$	0, 1	0, 1
	C	0, 1	1, 0	0, 1
	R	0, 1	0, 1	β, $1-\beta$

Figure 1: The penalty kick in soccer

a) (5) Is there a pure strategy Nash equilibrium for this game? Justify briefly.

b) (20) Let q_L, q_C, q_R be the probabilities that the striker plays L, C, R respectively. Let p_L, p_C, p_R be the probabilities that the goalie plays L, C, R respectively. Find a mixed-strategy Nash equilibrium (MSNE) for which both players will play each of the three strategies with strictly positive probability. Briefly explain your steps.

c) (10) In the MSNE that you have found above, which of the three strategies will be played with lowest probability by the goalie? Interpret briefly why.

d) (5) Let $\alpha = 0.4$ and $\beta = 0.6$. Calculate the probability that the striker will score a goal under the MSNE that you found. Briefly explain your procedure.

[1] This is a modified version of problem 7.13 in Jehle and Reny (2011).

4. (40 points) Uncertainty and the VNM utility function[2]

Consider the quadratic VNM utility function $U(w) = a + bw + cw^2$.

a) (10) What restrictions if any must be placed on parameters a, b, and c for this function to display risk aversion?

b) (10) Over what domain of wealth can a quadratic VNM utility function be defined?

c) (20) Given the gamble

$$g = ((1/2) \circ (w + h), (1/2) \circ (w - h)),$$

show that $CE < E(g)$ and that $P > 0$.

[2]Problem 2.25 in Jehle and Reny (2011).