August 4, 2015

ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa

final exam
Time allotted: 3 hours
Professor: Louis Hotte

NB This questionnaire has 3 pages. Some formulae are provided in the APPENDIX (that you may or may not need).

1. (25 points) Existence of a VNM Utility Function on *G*

THEOREM: *Let preferences* $\succcurlyeq$ *over gambles in* G *satisfy axioms G1 to G6 (see Appendix). Then there exists a utility function* $u : G \rightarrow \Re$ *representing* $\succcurlyeq$ *on* G*, such that* u *has the expected utility property.*

Demonstrate the first part only of this theorem, i.e., demonstrate that "there exists a utility function $u : G \rightarrow \Re$ representing $\succcurlyeq$ on G". (You need NOT show that u has the expected utility property.)

2. (25 points) Cost functions

a) (5) Give the definition of a cost function in both words and formally.

b) (20) A real-valued function is called *superadditive* if $f(z1 + z2) \geq f(z1) + f(z2)$. Show that every cost function is superadditive in input prices. Use this to prove that the cost function is non-decreasing in input prices without requiring it to be differentiable.

3. (25 points) Nash equilibrium and price competition

Firm A and firm B are two textile companies that produce the same silk cloth. Firm A has a technology that uses only labor (L) as an input. Its production function is

$$Q_A = L.$$

Firm B has a technology that uses both capital (K) and labor (L). Its production function is

$$Q_B(K, L) = 2\sqrt{KL}$$

. Let r and w represent the capital rental rate and the wage rate, and assume that both companies are price-takers in the input markets.

Suppose that the market demand for silk cloth is given by

$$Q = 120 - P$$

where P is the prevailing market price of cloth.

Both firms are profit maximizers. The two firms set the prices of their cloth (P_A and P_B) simultaneously, where P_A and P_B can take any integer value between 0 and 120 (no fractional prices are permitted). If $P_A \neq P_B$, consumers buy only from the lower-price firm; if $P_A = P_B$ they divide their purchases equally between the two firms.

Find Nash equilibrium prices when $w = 10$ and $r = 40$. Explain briefly but clearly all the steps in your work.

4. (25 points) Consumer choice

Lu consumes only one good: rice. Let x be the amount of rice he consumes, p the price of rice, and w his wealth. His utility function is given by

$$u(x) = 20x - x^2, \ \forall \ x \geq 0.$$

1. Illustrate the graph of his utility function.
2. Describe the indifference curve for $u = 64$. Is his utility function quasiconcave? Explain.
3. Assume that Lu must consume whatever he buys [no free disposal]. Find demand for $p, w \geq 0$. Explain your reasoning.
4. Find Lu's indirect utility function $v(p, w)$.

APPENDIX: Some axioms, definitions and properties

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - x_j \frac{\partial x_i}{\partial y}$$

$$x_i = -\frac{\partial v/\partial p_i}{\partial v/\partial y}$$

$$p_i(\mathbf{x}) = \frac{\partial u/\partial x_i}{\sum_{j=1}^{n} x_j(\partial u/\partial x_j)}$$

Axioms of consumer choice:

1. Completeness. Either $\mathbf{x}^1 \succsim \mathbf{x}^2$ or $\mathbf{x}^2 \succsim \mathbf{x}^1$.
2. Transitivity. If $\mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^3$, then $\mathbf{x}^1 \succsim \mathbf{x}^3$.
3. Continuity. $\forall \mathbf{x}$ the sets $\succsim (\mathbf{x})$ and $\precsim (\mathbf{x})$ are closed.
4. Strict monotonicity. $\forall \mathbf{x}^0, \mathbf{x}^1$, if $\mathbf{x}^0 \geq \mathbf{x}^1$ then $\mathbf{x}^0 \succsim \mathbf{x}^1$, while if $\mathbf{x}^0 \gg \mathbf{x}^1$ then $\mathbf{x}^0 \succ \mathbf{x}^1$.
5. Convexity. If $\mathbf{x}^0 \succsim \mathbf{x}^1$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in [0,1]$.
6. Strict convexity. If $\mathbf{x}^1 \neq \mathbf{x}^0$ and $\mathbf{x}^1 \succsim \mathbf{x}^0$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in (0,1)$.

Axioms of choice under uncertainty:

1. Completeness. Either $g \succsim g'$ or $g' \succsim g$.
2. Transitivity. If $g \succsim g'$ and $g' \succsim g''$, then $g \succsim g''$.
3. Continuity. $\exists \alpha \in [0,1]$ s.t. $g \sim (\alpha \oplus a_1, (1-\alpha \oplus a_n))$.
4. Monotonicity. $\forall \alpha, \beta \in [0,1]$, $(\alpha \oplus a_1, (1-\alpha \oplus a_n)) \succsim (\beta \oplus a_1, (1-\beta \oplus a_n))$ iff $\alpha \geq \beta$.
5. Substitution. If $g = (p_1 \oplus g_1, ..., p_k \oplus g_k)$ and $h = (p_1 \oplus h_1, ..., p_k \oplus h_k)$ and $h_i \sim g_i, \forall i$, then $h \sim h$.
6. Reduction to simple gambles. ...

Arrow-Pratt measure of risk-aversion:

$$R_a(w) \equiv \frac{-u''(w)}{u'(w)}.$$

Some definitions:

1. Quasiconcave function. $f : D \to R$ is quasiconcave iff, for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in D, $f(\mathbf{x}^t) \geq \min[f(\mathbf{x}^1), f(\mathbf{x}^2)]$ for all $t \in [0,1]$.
2. Concave function. $f : D \to R$ is a concave function if for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in D, $f(\mathbf{x}^t) \geq tf(\mathbf{x}^1) + (1-t)f(\mathbf{x}^2)$ for all $t \in [0,1]$.
3. Convex set. $S \in \Re^n$ is a convex set if for all $\mathbf{x}^1 \in S$ and $\mathbf{x}^2 \in S$, we have $t\mathbf{x}^1 + (1-t)\mathbf{x}^2 \in S$ for all $t \in [0,1]$.