April 27, 2015

ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa

final exam
Time allotted: 3 hours
Professor: Louis Hotte

NB This questionnaire has ? pages. Some formulae are provided in the APPENDIX (that you may or may not need).

1. (25 points) The Core of an Exchange Economy For this question, we consider only an endowment economy, i.e. without production.

a) (5) Define the core of an exchange economy, denoted $C(\mathbf{e})$, where $\mathbf{e}$ is the economy's endowment vectors.

b) (5) Define a Walrasian equilibrium

c) (10) Let $W(\mathbf{e})$ be the set of Walrasian equilibrium allocations. Show that $W(\mathbf{e}) \subset C(\mathbf{e})$. (Hint: Proceed by contradiction.)

d) (5) Discuss, in words only, the implications of that result in comparison to a barter exchange economy.

2. (25 points) Imperfect Competition Answer **only one** of the following two questions:

I) (25) Suppose an industry with a duopoly in which the firms compete in Cournot-Nash fashion. Each duopolist have constant average and marginal costs, denoted c_1 and c_2 for firms 1 and 2 respectively. These costs, are different for each firm: Firm 1 has lower costs than firm 2, i.e. $c_1 < c_2$. Show that firm 1 will have greater profits and produce a greater share of market output than firm 2 in the Nash equilibrium.

II) A monopolist faces linear demand $p = \alpha - \beta q$ and has cost $C = cq + F$, where all parameters are positive, $\alpha > c$, and $(\alpha - c)^2 > 4\beta F$.

   a) (10) Solve for the monopolist's output, price, and profits.

   b) (5) Calculate the deadweight loss and show that it is positive.

   c) (10) If the government requires this firm to set the price that maximizes the sum of consumer and producer surplus, and to serve all buyers at that price, what is the price the firm must charge? Show that the firm's profits are negative under this regulation, so that this form of regulation is not sustainable in the long run.

3. (25 points) Prove only one of the following two theorems:

I) Theorem (The Law of Demand) Show that a decrease in the own price of a normal good will cause quantity demanded to increase. If an own price decrease causes a decrease in quantity demanded, the good must be inferior.

II) Theorem (Walras' Law). Consider an exchange economy with endowments only (no production). The set of consumers is given by $\{1, 2, ..., I\}$ and there are n goods. Assume that utility u^i is continuous, strongly increasing, and strictly quasiconcave on $\Re^n_+$. If $p \gg 0$, and if all but one market clears, then the last market must clear also.

4. (25 points) Risk Aversion Suppose that Scarlet has a [Bernoulli] utility function given by $u = log(w)$, where w is her wealth in money.

a) (10) Suppose Scarlet has wealth w and is offered a bet with a 50/50 chance of winning or losing βw dollars, where $0 < \beta < 1$. Write down the lottery that Scarlet faces if she accepts the bet. Find the expected utility of the lottery and its certainty equivalent c.

b) (5) Use your answer in (a) to explain why Scarlet displays decreasing absolute risk aversion. (NB I do not want you to use the Arrow-Pratt measure here. Use your result in (a).)

c) (10) How does Scarlet's willingness to pay to avoid the bet vary as a proportion of her wealth? What does this say about her degree of risk aversion relative to her wealth?

APPENDIX: Some axioms, definitions and properties

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - x_j \frac{\partial x_i}{\partial y}$$

$$x_i = -\frac{\partial v/\partial p_i}{\partial v/\partial y}$$

$$p_i(\mathbf{x}) = \frac{\partial u/\partial x_i}{\sum_{j=1}^n x_j(\partial u/\partial x_j)}$$

Axioms of consumer choice:

1. Completeness. Either $\mathbf{x}^1 \succsim \mathbf{x}^2$ or $\mathbf{x}^2 \succsim \mathbf{x}^1$.
2. Transitivity. If $\mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^3$, then $\mathbf{x}^1 \succsim \mathbf{x}^3$.
3. Continuity. $\forall \mathbf{x}$ the sets $\succsim (\mathbf{x})$ and $\precsim (\mathbf{x})$ are closed.
4. Strict monotonicity. $\forall \mathbf{x}^0, \mathbf{x}^1$, if $\mathbf{x}^0 \geq \mathbf{x}^1$ then $\mathbf{x}^0 \succsim \mathbf{x}^1$, while if $\mathbf{x}^0 \gg \mathbf{x}^1$ then $\mathbf{x}^0 \succ \mathbf{x}^1$.
5. Convexity. If $\mathbf{x}^0 \succsim \mathbf{x}^1$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in [0,1]$.
6. Strict convexity. If $\mathbf{x}^1 \neq \mathbf{x}^0$ and $\mathbf{x}^1 \succsim \mathbf{x}^0$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in (0,1)$.

Axioms of choice under uncertainty:

1. Completeness. Either $g \succsim g'$ or $g' \succsim g$.
2. Transitivity. If $g \succsim g'$ and $g' \succsim g''$, then $g \succsim g''$.
3. Continuity. $\exists \alpha \in [0,1]$ s.t. $g \sim (\alpha \oplus a_1, (1 - \alpha \oplus a_n))$.
4. Monotonicity. $\forall \alpha, \beta \in [0,1]$, $(\alpha \oplus a_1, (1-\alpha \oplus a_n)) \succsim (\beta \oplus a_1, (1-\beta \oplus a_n))$ iff $\alpha \geq \beta$.
5. Substitution. If $g = (p_1 \oplus g_1, ..., p_k \oplus g_k)$ and $h = (p_1 \oplus h_1, ..., p_k \oplus h_k)$ and $h_i \sim g_i, \forall i$, then $h \sim h$.
6. Reduction to simple gambles. ...

Arrow-Pratt measure of risk-aversion:

$$R_a(w) \equiv \frac{-u''(w)}{u'(w)}.$$

Some definitions:

1. Quasiconcave function. $f : D \to R$ is quasiconcave iff, for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in D, $f(\mathbf{x}^t) \geq \min[f(\mathbf{x}^1), f(\mathbf{x}^2)]$ for all $t \in [0,1]$.
2. Concave function. $f : D \to R$ is a concave function if for all $\mathbf{x}^1$ and $\mathbf{x}^2$ in D, $f(\mathbf{x}^t) \geq tf(\mathbf{x}^1) + (1-t)f(\mathbf{x}^2)$ for all $t \in [0,1]$.
3. Convex set. $S \in \Re^n$ is a convex set if for all $\mathbf{x}^1 \in S$ and $\mathbf{x}^2 \in S$, we have $t\mathbf{x}^1 + (1-t)\mathbf{x}^2 \in S$ for all $t \in [0,1]$.