

Microeconomic Theory IV
ECO 6122
Mid-Term Exam
October 31, 2006

NB : Neither documentation, nor calculator are necessary. Mathematical symbols are the same as the ones used in class. This questionnaire has two pages.

Part A. Consumer Theory: True, False or Uncertain. (20 points).
Respond to the following 4 questions. Justify your response.

1. The transitive property of preferences implies that the consumer prefers average to extreme basket of goods.
2. The matrix of substitution terms $\partial h_j(p,u) / \partial p_i$ is negative semi-definite because the expenditure function is quasi-convex.
3. Changes in consumer surplus (ΔCS) is equivalent to Compensating variation (CV) for measuring changes in welfare.

Part B. Consumer Theory Problems. Respond to the following 3 questions. (30 points)

4. Assume that a consumer has the following expenditure function $e(p_1, p_2, u) = p_1^a p_2^{1-a} u$ and budget constraint $p_1 x_1 + p_2 x_2 = m$.
 - a. Derive the Hicksian demands $h(p,u)$.
 - b. Find the indirect utility function $v(p,m)$.
 - c. Derive the Marshallian demand functions $x(p,m)$
 - d. Find the corresponding money metric direct utility function $m(p,x)$.
5. A person has an expected utility of the form $u(w) = \sqrt{w}$. He initially has wealth of \$9. He has a lottery ticket that will be worth \$27 with probability of 0.6 and will be worth \$0 with probability of 0.4.
 - a. What is his expected utility?
 - b. Would that person accept to sell his lottery ticket at a price of \$16?

Part C. Producer Theory. Review Question. (20 points)

6a) (3 pts) What is the definition of a profit function?

6b) (12 pts) Is the profit function convex or concave? Show why with either a rigorous proof or an intuitive one. (NB You get a 2 point bonus if you give the complete rigorous proof.)

6c) (5 pts) What are the properties of the technology that are required in order to make the proof in question 6b)?

Part D. Producer Theory. Problems. (30 points)

7. (15 pts) A firm produces one type of output, $y \geq 0$, with two types of inputs $x_1 \geq 0$ and $x_2 \geq 0$. Suppose that the prices of both inputs increase by $dw_1 > 0$ and $dw_2 > 0$ respectively. Can you find conditions under which the profit maximizing output will increase nonetheless? Interpret your results.

8. (15 pts) A utility produces electricity to meet the demands of a city. The price it can charge for electricity is fixed and it must meet all demand at that price. It turns out that the amount of electricity demanded is always the same over every 24-hour period, but demand differs from day (6:00 a.m. to 6:00 p.m.) to night (6:00 p.m. to 6:00 a.m.). During the day, 4 units are demanded, whereas during the night only 3 units are demanded. Total output for each 24-hour period is thus always equal to 7 units. The utility produces electricity according to the production function

$$y_i = (KF_i)^{1/2}, \quad i = \text{day, night},$$

where K is the size of the generating plant, and F_i is tons of fuel. The firm must build a single plant; it cannot change plant size from day to night. If a unit of plant size costs w_k per 24-hour period and a ton of fuel costs w_f , what size plant will the utility build?