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ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa
Final exam
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This exam has 3 pages. Some formulae and definitions are provided in the Appendix.

1. (25 points) A city government considers renovating its water treatment facilities which will allow a reduction in the price of water from p_0 to p_1. Let q be the quantity of water being consumed and m be the amount spent on *all other goods* by the consumer.

a) **(15)** Using the concept of *compensating variation*, determine graphically what is the maximum that the consumer is willing to pay for the new treatment facilities.

b) **(5)** Show graphically how the compensating variation differs from the consumer surplus.

c) **(5)** Discuss what is the problem with the use of compensating variation in practice and what can be done about it.

2. (25 points) An individual with initial wealth w_0 and VNM utility function $u(\cdot)$ must decide whether and for how much to insure his car. The probability that he will have an accident and incur a loss of L in damages is $\alpha \in (0, 1)$. If he insures himself for an amount x, then his loss in the case of an accident is limited to $L - x$.

Each unit of insurance costs q. The total cost of purchasing x units is thus qx. For the insurance company, the expected profit from selling x units is $E[\pi] = \alpha(qx - x) + (1 - \alpha)(qx)$.

a) **(5)** Show mathematically that if the insurance price q is *actuarially fair* (that is, $E[\pi] = 0$), then the risk-averse individual will choose to be fully insured.

b) **(5)** Show graphically that with an actuarially fair insurance and full insurance, the expected utility of the risk-averse individual is above the one that he would achieve without any insurance.

c) **(5)** Show graphically that even if the price of insurance were slightly above the actuarially fair price, a fully insured individual can still be better off than without any insurance at all.

d) **(5)** Show mathematically that if the insurance price q is NOT actuarially fair (that is, $E[\pi] > 0$), then the risk-averse individual will choose NOT to be fully insured.

e) **(5)** In case d), derive an inequality condition under which the individual will choose no insurance at all. Explain how this inequality is affected by the degree of risk-aversion of the individual.

3. (25 points) Two cabbage farmers meet every Saturday morning at the town market. Every time, they must decide on the cabbage price. Assuming a repeated, symmetric duopoly, the **per-week** payoff to each firms is π^m if they both choose the high price for cabbage that maximizes their joint profits and $\pi^c = 0$ if they both choose a low price. The maximum one-day payoff that one farmer can get if the other chooses the high price is π^d. We have $\pi^d > \pi^m > 0$.

a) **(15)** Assume that both players adopt the trigger strategy of reverting to the low price if either player defects from the joint profit maximizing high price in the past. The farmers' weekly discount rate is r. Find the maximum value of the discount rate under which the duopoly can indefinitely sustain a cooperation equilibrium with joint profit maximization.

b) **(5)** Suppose that a new town by-law restricts the market day to once a month (every four week) instead of once a week. As a result, the discount factor between each period that the farmers meet is now $1/(1+r)^4$. (Assume that nothing else is affected.) Determine how this change affects the possibility of cooperation between farmers and discuss.

c) **(5)** Go back to case a) where farmers meet every week. Suppose that demand for cabbage grows weekly at a regular rate, such that future profits now depend on time as follows:

$$\pi^d(t) = (1+g)^t \pi^d, \ \pi^m(t) = (1+g)^t \pi^m \text{ and } \pi^c(t) = 0.$$

Determine how this change affects the possibility of cooperation between farmers and discuss.

4. (25 points) In an economy with two consumption goods ($i = 1, 2$), the utility function of an individual can be represented as $u(\vec{x}) = (x_1 - a_1)^\alpha x_2^{1-\alpha}$, with $\alpha \in (0, 1)$ and $a_1 \geq 0$. Parameter a_1 is referred to as the minimum *subsistence consumption* level for good 1. Given total income level y, income in excess of the subsistence level is thus $y - p_1 a_1$ and is referred to as *discretionary income.*

a) **(10)** Derive the indirect utility function for this individual and show that it is linear in discretionary income.

b) **(10)** Derive the expenditure function and show that it is linear in utility level u.

c) **(5)** Explain, either intuitively or rigourously, whether an expenditure function should be concave or convex with respect to p_1. Verify that the one that you derived above respects the right concavity-convexity property.

APPENDIX

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - x_j \frac{\partial x_i}{\partial y}$$

$$x_i = -\frac{\partial v / \partial p_i}{\partial v / \partial y}$$

$$p_i(\mathbf{x}) = \frac{\partial u / \partial x_i}{\sum_{j=1}^{n} x_j (\partial u / \partial x_j)}$$

Axioms of consumer choice:

1. Completeness. Either $\mathbf{x}^1 \succcurlyeq \mathbf{x}^2$ or $\mathbf{x}^2 \succcurlyeq \mathbf{x}^1$.
2. Transitivity. If $\mathbf{x}^1 \succcurlyeq \mathbf{x}^2$ and $\mathbf{x}^2 \succcurlyeq \mathbf{x}^3$, then $\mathbf{x}^1 \succcurlyeq \mathbf{x}^3$.
3. Continuity. $\forall \mathbf{x}$ the sets $\succcurlyeq (\mathbf{x})$ and $\precsim (\mathbf{x})$ are closed.
4. Strict monotonicity. $\forall \mathbf{x}^0, \mathbf{x}^1$, if $\mathbf{x}^0 \geq \mathbf{x}^1$ then $\mathbf{x}^0 \succsim \mathbf{x}^1$, while if $\mathbf{x}^0 \gg \mathbf{x}^1$ then $\mathbf{x}^0 \succ \mathbf{x}^1$.
5. Convexity. If $\mathbf{x}^0 \succsim \mathbf{x}^1$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in [0,1]$.
6. Strict convexity. If $\mathbf{x}^1 \neq \mathbf{x}^0$ and $\mathbf{x}^1 \succsim \mathbf{x}^0$, then $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0, \forall t \in (0,1)$.

Axioms of choice under uncertainty:

1. Completeness. Either $g \succcurlyeq g'$ or $g' \succcurlyeq g$.
2. Transitivity. If $g \succcurlyeq g'$ and $g' \succcurlyeq g''$, then $g \succcurlyeq g''$.
3. Continuity. $\exists \alpha \in [0,1]$ s.t. $g \sim (\alpha \oplus a_1, (1 - \alpha \oplus a_n))$.
4. Monotonicity. $\forall \alpha, \beta \in [0,1]$, $(\alpha \oplus a_1, (1 - \alpha \oplus a_n)) \succcurlyeq (\beta \oplus a_1, (1 - \beta \oplus a_n))$ iff $\alpha \geq \beta$.
5. Substitution. If $g = (p_1 \oplus g_1, ..., p_k \oplus g_k)$ and $h = (p_1 \oplus h_1, ..., p_k \oplus h_k)$ and $h_i \sim g_i, \forall i$, then $h \sim h$.
6. Reduction to simple gambles. ...

Arrow-Pratt measure of risk-aversion:

$$R_a(w) \equiv \frac{-u''(w)}{u'(w)}.$$