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ECO 6122: Microeconomic Theory IV
Economics Department
University of Ottawa
Final exam
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NB This exam has 3 pages.

Part A. (50 points)

Question 1: Duopoly and game theory (25 points) Consider two producers with cost functions $c_1(y_1) = 10y_1$ and $c_2(y_2) = 100 + 22y_2$, respectively. Demand is given by $Y \equiv y_1 + y_2 = 1000 - 10p$. (Note that firms are not identical. You should neither assume, nor expect identical equilibrium outcomes between the two firms.)

1a) (6 pts) Compute the Cournot-Nash equilibrium price, quantities, consumer surplus and producer surplus. (NB Cournot means that firms compete in quantities. Derive each firm's reaction function.)
1b) (6 pts) Compute the Bertrand-Nash equilibrium price, quantities, consumer surplus and producer surplus. (NB Bertrand means competition in prices. The lowest price firm gets the whole market demand. If prices are equal, demand is split equally between the two firms.)
1c) (6 pts) Compute the perfect competition equilibrium prices, quantities, consumer surplus and producer surplus. (NB Make sure to begin by defining precisely what is meant by *perfect competition.*)
1d) (7 pts) Compare the Cournot, Bertrand and perfect competition competition outcomes in terms of social welfare and interpret your results.

Question 2: Producer theory (25 points) A manufacturing firm has a production function $Q = \alpha L^\beta K^\lambda$, where Q is output, L is labor and K is capital. α is a parameter that represents total factor productivity. Output, labor and capital prices are denoted p, w and r respectively. The producer is assumed to maximize profits.

1a) (9 pts) Find the first order conditions for profit maximization.
Given your first-order conditions, show that for total revenues to exceed total costs, it must be the case that $\beta + \lambda < 1$. Interpret. (Hint: Once you have set-up the first-order conditions, this is very straightforward to show in two or three lines. Do not attempt to solve explicitly

for L^*, K^* and profits to answer this.)

1b) (8 pts) From now on, assume that $\lambda = \beta = 0.4$. Derive the input demand, output supply and profit functions.

1c) (8 pts) Verify that the sign of the first and second derivatives of the profit function with respect to the wage rate w correspond to what you would expect. (NB You must justify what you would expect in simple economic terms.)

Part B.

a) True, False or Uncertain (15 points).

Respond to the following 3 questions. Justify your response.

1. When the expected utility of a gamble is higher than the utility of the expected value of the gamble, the consumer is considered risk lover.
2. The Walras Law applies to any price in an economy as long as the preferences of the consumers are transitive.
3. All pareto optimal allocations are in the core of the economy.

b) General Equilibrium Problems. Respond to the following 3 questions. (35 points)

4. (10 points). Consider a two-good, one representative agent market economy. The production of good 1 is 10. The production of good 2 is 15. The representative agent has the following utility function with respect to good 1 and 2:: $U(x_1, x_2) = (x_1^{\rho} + x_2^{\rho})^{1/\rho}$, where x_1 and x_2 are the consumption of good 1 and good 2 respectively and $0 \neq p < 1$.
   (a) Assume that the income of the representative agent is *M*, derive the marshallian demand functions for good 1 and for good 2.
   (b) Assume that for this economy the equilibrium exists. Can we claim that this equilibrium is also unique?

5. (10 points) Assume an exchange economy composed of two consumers *A* and *B*. The utility function of consumer A is $U_A(x_1^A, x_2^A) = (x_1^{.5} x_2^{.5})$ while the utility function of consumer B is $U_B(x_1^B, x_2^B) = (x_1^{.8} x_2^{.2})$. The total endowment of good 1 and good 2 is $\omega = (25{,}10)$.
   (a) Find one optimal Pareto allocation for this exchange economy.
   (b) Illustrate the Pareto allocation in an Edgeworth Box

6. (15 points) Assume a Robinson Crusoe economy with a Utility function $U_A(x, R) = (x^{.4} R^{.6})$, where *x* is the consumption good and *R* is leisure. Assume that the technology used to produce *x* is $x = L^{.5}$, where *L* is labour. Robinson also faces the resource constraint $L+R=1$.
   (a) Find the demand function for labour.
   (b) Find the supply of *x*.
   (c) Find the profit function.
   (d) Find the supply of labour.
   (e) Find the equilibrium prices of this economy.
   (f) Find the equilibrium quantities of *x*, *R*, and *L*.