SOLUTIONS TO EXERCISES
CHAPTER 8: TECHNOLOGY

1) Do problems 1 and 2 in Weil chap 8.

2) **Slowdown in productivity growth**
Consider the following two scenarios:
   i) The rate of technological progress declines forever.
   ii) The savings rate declines forever.
   a) Analyse graphically, what is the impact of each of these scenarios on economic growth in the next five years (short run)?
   b) Over the next five decades (long run)?
   Discuss the effects on both growth rates and output levels.

3) **Steady-state output and technological progress**
Suppose that the economy's production function is
$$Y = AK^{\alpha}L^{1-\alpha} = K^{\alpha}(eL)^{1-\alpha}, \text{ with } \alpha = 1/2$$
where A denotes TFP and $e \equiv A^2$. Based on the notation used in class, we have $\gamma = 16\%$, $\delta = 10\%$, $n = 2\%$ and $\hat{e} = 4\%$ per year.
   a) Find the steady-state values of
      i) Capital stock per effective worker
      ii) Output per effective worker
      iii) Growth rate of output per effective worker
      iv) Growth rate of output per worker
      v) Growth rate of output
      vi) Output level per worker as given by $y_t^{ss} = e_t y_e^{ss}$.
   b) Suppose that the rate of technological progress $\hat{e}$ jumps to 8%. Recompute the answers to a) and discuss your results.
   c) Suppose that $\hat{e}$ is still equal to 4% but that worker population growth is now $n = 6\%$. Recompute the answers to a).
   d) Compare the welfare of the workers in a), b) and c) in terms of level of income per worker. Discuss.

Problem 1:

a) Nonrival. Nonexcludable. One's consumption of National Defense does not diminish another's consumption of National Defense, and within a given country's borders, it is difficult to selectively exclude others from consuming National Defense.
b) Rival. Excludable. Once a cookie is consumed, no one can consume that cookie. Furthermore, one can easily prevent another from consuming the cookie.
c) Non-Rival. Excludable. My authorized use of a website does not diminish another's use of the same website. However, this good is excludable because a password is required, and so only those selected can access the website.
d) Rival. Nonexcludable. The consumption of a piece of fruit ensures that no other person can consume that same piece of fruit. However, because the fruit grows in a public square, anyone is able to consume the fruit.

Problem 2:

The advantage of a patent for a life-saving drug is that by granting a monopoly to its discoverer, it provides incentives for firms to engage in costly R&D activities in order to continuously discover better life-saving drugs.

The drawback, of course, is that monopoly profits imply higher prices, which means that some poorer individuals may not be able to afford those new drugs.

There are various possible solutions to this problem.

A partial solution is to give the patent an expiry date. In Canada, patents have an expiry date of 20 years (and similarly for much of the developed world). This means that all medicine discovered over 20 years ago are generally quite affordable all over the world.

In Québec and other parts of the developed world, a solution has been to introduce mandatory participation in drug insurance schemes.

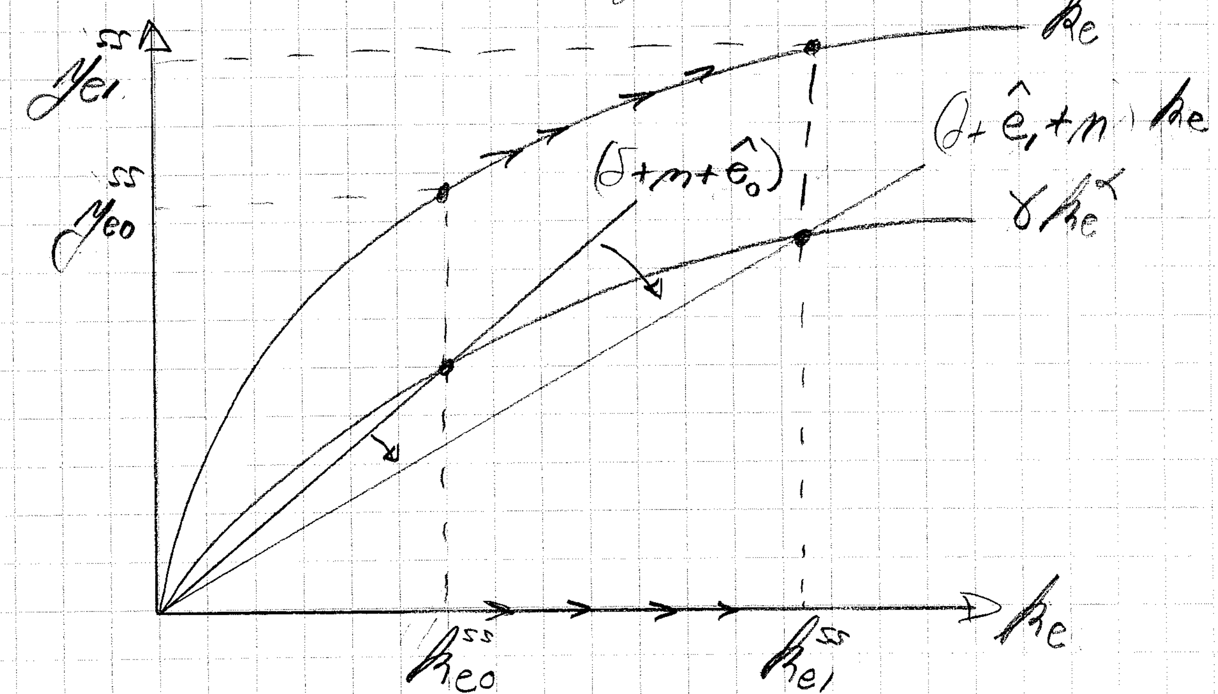
The most difficult issues are with respect to the poor living in less-developed economies who cannot generally be covered by insurance. A recent solution imposed on drug companies has been to "force" them to supply the drugs at lower prices in those countries. (Drug companies were reluctant, arguing that the cheaper drugs would be sent back to richer countries through the black market. One wonders how important this effect can be.)

Others have proposed that less-developed economies should not provide patent protection. The problem is that this sharply reduces incentives for R&D activities aimed at diseases that are not present in the developed world.

2) SLOWDOWN IN PRODUCTIVITY GROWTH

i) Slowdown in productivity growth

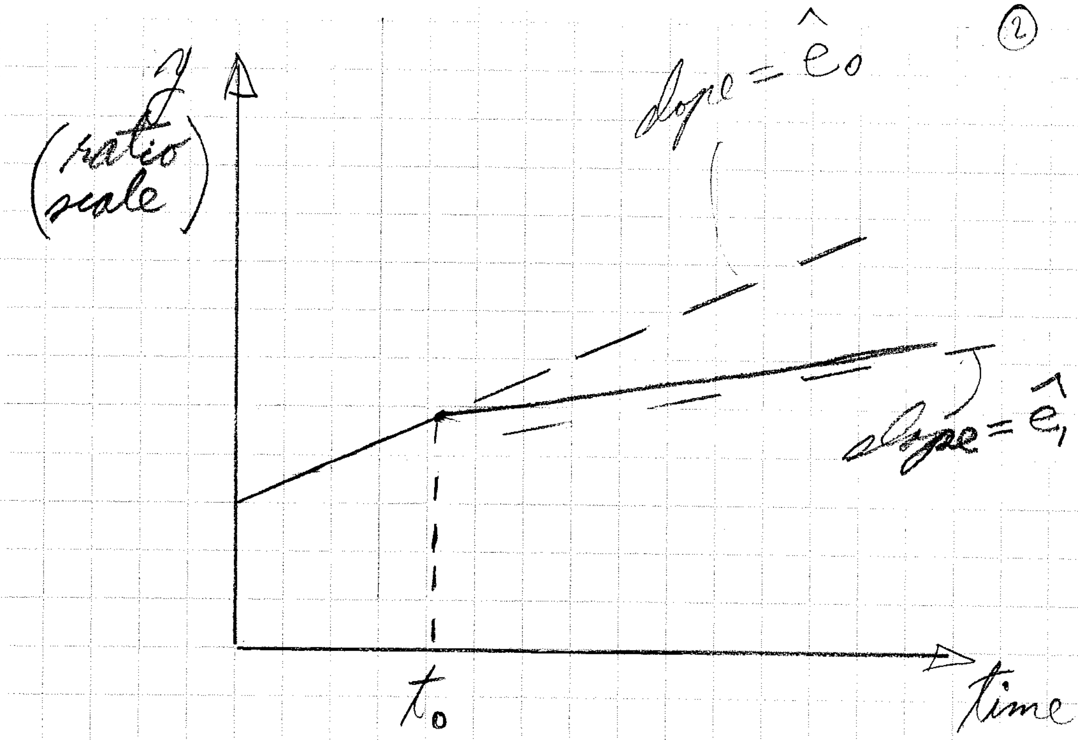
→ Slowdown in technological progress at time t_0 and permanent:



$\hat{e}_1 < \hat{e}_0$

$k_e \equiv \frac{K}{eL}$

A permanent drop in T.P. from $\hat{e}_0$ to $\hat{e}_1$ leads to a gradual increase in capital per effective worker from k_{e0}^{ss} to k_{e1}^{ss}. This is because future values of e will be lower. Output per *effective* worker will also go up. This may look like an improvement, but it is not. Smaller growth of e means that its future values are smaller than in the initial trajectory.

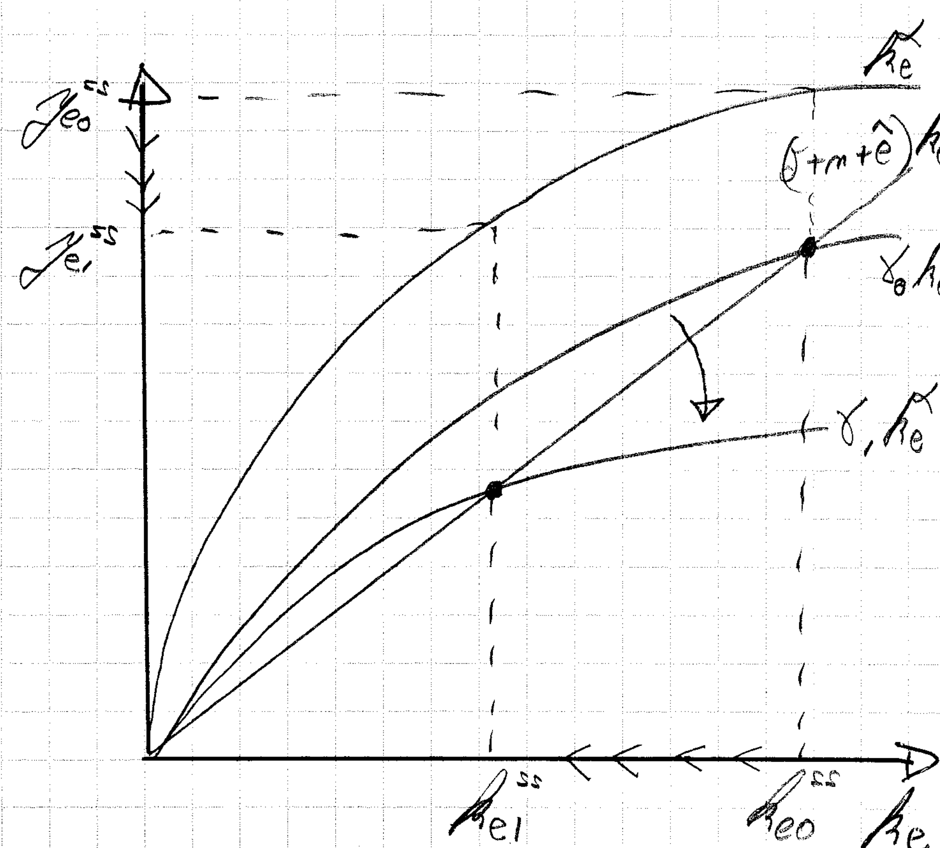


Assume that the economy is initially in a steady-state growth path and that T.P. drops from $\hat{e}_0$ to $\hat{e}_1$ at time t_0. The growth rate of income per capita will slow down and converge towards $\hat{e}_1$ in the long-run.

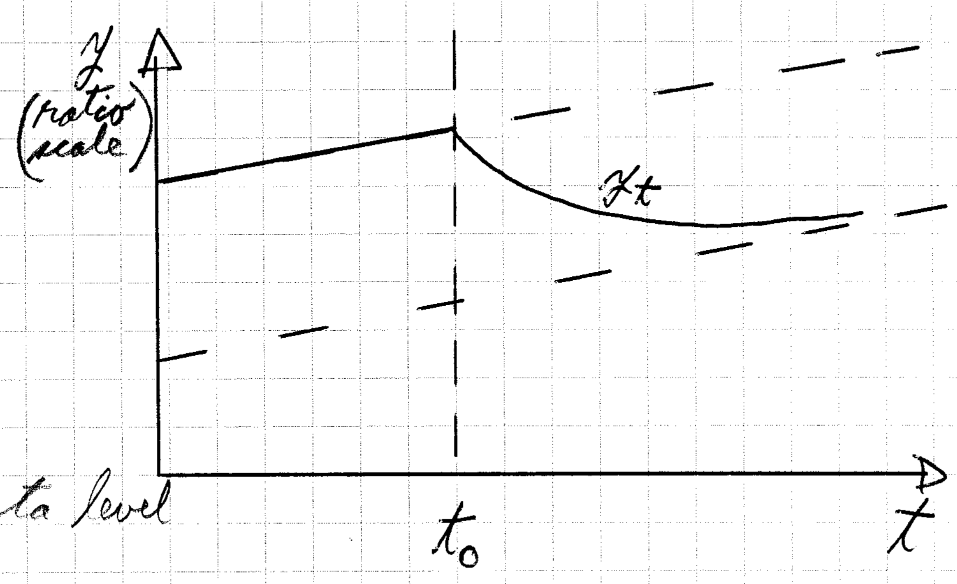
Over 5 years, both output level and output growth are lower. The same is true over 5 decades.

ii) Permanent decline in the savings rate

At t_0, γ drops from γ_0 to γ_1, with $\gamma_1 < \gamma_0$.



In the short run, compared to the initial trajectory, income per capita drops in both level and growth rate.



In the long-run, income per-capita level is lower, but its growth rate is the same.

N.B. $y_t^{ss} = e_t\, y_{et}^{ss} = e_t\left(\dfrac{\gamma}{\delta+n+\hat{e}}\right)^{\frac{\alpha}{1-\alpha}} \Rightarrow \Delta^-\gamma \Rightarrow \Delta^- y_t^{ss}$

3) SS OUTPUT AND TECH. PROGRESS

a) In SS, we must have $\Delta k_e = 0$ where $k_e = \frac{K}{eL}$.

$$\Delta k_e = \gamma k_e^{1/2} - (\delta + n + \hat{e}) k_e = 0$$

$$\Rightarrow k_e^{ss} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^2 = \left(\frac{0.16}{0.1 + 0.02 + 0.04}\right)^2$$

i) $\Rightarrow \boxed{k_e^{ss} = 1}$

ii) $\Rightarrow y_e^{ss} = (k_e^{ss})^{1/2} = 1$

iii) $\hat{y}_e^{ss} = 0$ by definition of the SS since $\hat{k}_e^{ss} = 0$.

iv) $y_e = \frac{y}{e} \Rightarrow \hat{y}_e = \hat{y} - \hat{e}$

Since $\hat{y}_e^{ss} = 0$, we have $\hat{y}^{ss} = \hat{e} = 4\%$.
Per capita income grows at a rate of 4% per year.

v) $y_e = \frac{Y}{eL} \Rightarrow \hat{y}_e = \hat{Y} - \hat{e} - n$

In SS: $\hat{Y}^{ss} = \hat{e} + n = 4\% + 2\% = 6\%$

Aggregate output grows at a rate of 6% per year in the long run.

vi) $y_{et} = \frac{y_t}{e_t} \Rightarrow y_t = e_t \, y_{et}$

$$\Rightarrow y_t^{ss} = e_t \, y_e^{ss} = e_t \cdot 1$$

$$\Rightarrow \boxed{y_t^{ss} = e_t}$$

b) $\underline{\hat{e} = 8\%}$

$$\Rightarrow k_e^{ss} = \left(\frac{0.16}{0.1+0.02+0.08}\right)^2 = 0.64$$

$$\Rightarrow y_e^{ss} = (0.64)^{1/2} = 0.8$$

$$\Rightarrow \hat{y}_e^{ss} = 0$$

$$\Rightarrow \hat{y}^{ss} = 8\%$$

$$\hat{Y}^{ss} = 8\% + 2\% = 10\%, \quad \boxed{y_t^{ss} = 0.8 e_t}$$

The increase in growth of TP actually reduces capital stock and income level per effective worker in the long run. But this does not mean that income per capita is reduced. In fact, in the long run, it now grows at a higher rate of 8% per year, while aggregate output growth increases to 10%.

c) $\hat{e} = 4\%$, $n = 6\%$.

$$\Rightarrow k_e^{ss} = \left(\frac{0.16}{0.1+0.06+0.04}\right)^2 = 0.64$$

$$\Rightarrow y_e^{ss} = (0.64)^2 = 0.8$$

$$\hat{y}_e^{ss} = 0 \text{ by def.}$$

$$\Rightarrow \hat{y}^{ss} = 4\%$$

$$\hat{Y}^{ss} = 4\% + 6\% = 10\%.$$

$$\boxed{y_t^{ss} = 0.8\, e_t}$$

d) In the long run levels of income per capita are:

$y_{at} = e_{at}$ (for a))

$y_{bt} = 0.8\, e_{bt}$ (for b))

$y_{ct} = 0.8\, e_{ct}$ (for c))

where $e_{at} = e_{ct} < e_{bt}$.

Hence: $y_{at} > y_{ct}$

People are clearly worse off all else equal with a higher population growth even though aggregate output grows faster.

Also: $Y_{bt} > Y_{ct}$

Even though the SS output per effective worker are the same in b) and c), per capita income levels will be higher with higher TP growth than population growth, as would be expected.

Note, however, how the aggregate output growth is the same for both b) and c), though for different reasons.

Finally, we need more mathematical derivation to compare the magnitudes of y_t^a and y_t^b. It can be shown that $y_t^b > y_t^a$, as would be expected by intuition, i.e. a higher TP growth rate increases per capita income, all else equal.