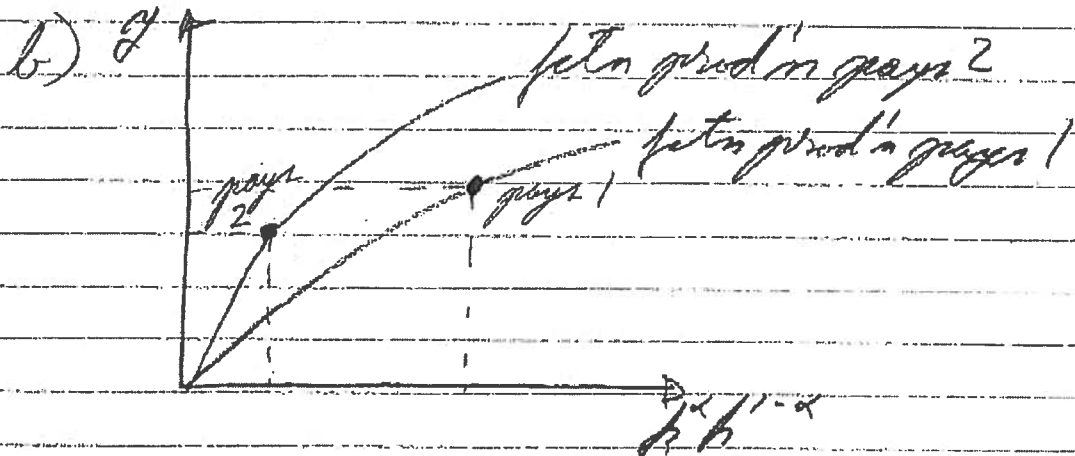
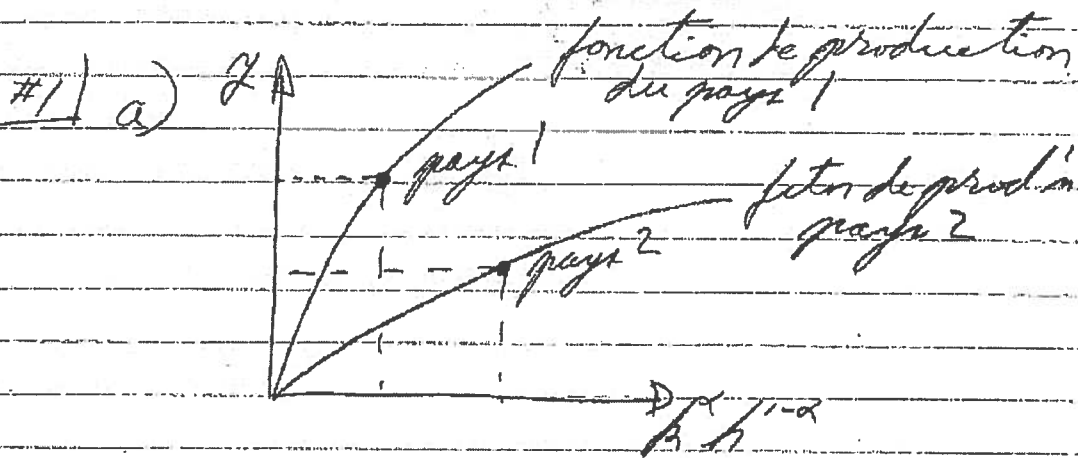


CHAPITRE 7: CHAPTER 7



#3 $y = A B^\alpha L^{1-\alpha}$, $\alpha = 1/2$

$$\Rightarrow A = \frac{y}{B^\alpha L^{1-\alpha}} = \frac{y}{\sqrt{B} \sqrt{L}}$$

a) $A_S = \frac{100}{\sqrt{100} \cdot \sqrt{25}} = \frac{100}{10 \cdot 5} = 2$

$A_F = \frac{200}{\sqrt{100} \cdot \sqrt{64}} = \frac{200}{10 \cdot 8} = 2,5$

$$b) \frac{y_S}{y_F} = \frac{A_S}{A_F} \frac{K_S^{1-\alpha} h_S^\alpha}{K_F^{1-\alpha} h_F^\alpha}$$

If quantities of human & physical K are the same, we have

$$\frac{y_S}{y_F} = \frac{A_S}{A_F} = \frac{2}{2.5} = 80\%$$

Income in Sylvania is 80% that of Fredonia.

c) If $A_S = A_F$, we have

$$\frac{y_S}{y_F} = \frac{K_S^{1-\alpha} h_S^\alpha}{K_F^{1-\alpha} h_F^\alpha} = \frac{\sqrt{100}^\alpha \sqrt{25}^\alpha}{\sqrt{100}^\alpha \sqrt{64}^\alpha} = \frac{50}{80} = 62.5\%$$

Income in Sylvania is now 62.5% that of Fredonia.

4) The numbers indicate the following:

- ① The gap in income has vanished between 1975 and 2010.
- ② The ratio of factor endowments has remained unchanged at $\frac{1}{2}$.

This suggests that country Z has made faster progress in its ability to use inputs in order to produce outputs, i.e. it exhibited faster productivity growth than country X.

7.5 We have

$$\frac{y_1}{y_2} = \left(\frac{A_1}{A_2} \right) \left(\frac{h_1^\alpha h_1^{1-\alpha}}{h_2^\alpha h_2^{1-\alpha}} \right) = \left(\frac{A_1}{A_2} \right) \left(\frac{h_1}{h_2} \right)^\alpha \left(\frac{h_1}{h_2} \right)^{1-\alpha}$$

For the Netherlands, this implies

$$\frac{y_N}{y_{USA}} = \frac{A_N}{A_{USA}} (0.81)^{1/3} (0.94)^{2/3} = \frac{A_N}{A_{USA}} (0.895)$$

$$\Rightarrow \frac{A_N}{A_{USA}} = \frac{0.87}{0.895} = 0.97$$

Factor accumulation differences explain that income in the Netherlands is 89.5% that of the USA. This leaves a factor of 97% for lower income in the Netherlands which is explained by lower productivity.

In other words, a lower factor endowment causes the Netherlands to be 10.5% poorer and its lower productivity causes it to be 3% poorer. Lower factor endowment therefore explains a much larger share of the income difference, i.e., it is 3.5 times more important.

For Paraguay, we have

$$\frac{y_P}{y_{USA}} = \frac{A_P}{A_{USA}} (0.094)^{1/3} (0.80)^{2/3} = \frac{A_P}{A_{USA}} (0.39)$$

$$\Rightarrow \frac{A_P}{A_{USA}} = \frac{0.097}{0.39} = 0.25$$

Lower factor endowment causes Paraguay to be $(1 - 0.39) = 61\%$ poorer than the USA. This leaves productivity to explain a lower income level by a factor of 75%. In this case, productivity is a more important contributor to the income difference than factor accumulation.

In Pakistan, we have

$$\frac{Y_{PK}}{Y_{USA}} = \frac{A_{PK}}{A_{USA}} \cdot (0.067)^{1/3} (0.65)^{2/3} = \frac{A_{PK}}{A_{USA}} (0.305)$$

$$\Rightarrow \frac{A_{PK}}{A_{USA}} = \frac{0.092}{0.305} = 0.302$$

Lower factor accumulation and lower productivity cause Pakistan to be 69.5% and 69.8% poorer respectively. They are both equal contributors to Pakistan's lower income level.

Note how the lower income of each of the three countries with respect to the USA, exhibits a different pattern. In the Netherlands case, it is almost all explained by a lower factor accumulation. In Paraguay's case, productivity plays a larger role but factor accumulation is still important. As for Pakistan, both factor accumulation and productivity are important and equally so.

7.6

Argentina:

$$\begin{aligned}\hat{A} &= \hat{y} - \alpha \hat{K} - (1-\alpha) \hat{H} \\ &= 0.66 - \frac{1}{3}(0.31) - \frac{2}{3}(0.52) = \\ &= 0.66 - 0.45\end{aligned}$$

$= 0.21\%$: growth rate of productivity
 0.45% is the growth rate due to
 factor accumulation

Uruguay:

$$\alpha \hat{K} + (1-\alpha) \hat{H} = \frac{1}{3}(1.83) + \frac{2}{3}(0.51) = 0.95$$

In Uruguay, factor accumulation
 causes income to grow by 0.95%
 annually. This leaves the following
 residual growth caused by productivity:

$$\hat{A} = \hat{y} - [\alpha \hat{K} + (1-\alpha) \hat{H}] = 1.82 - 0.95 = 0.87\%$$

Panama:

$$\alpha \hat{K} + (1-\alpha) \hat{H} = \frac{1}{3}(0.9) + \frac{2}{3}(0.84) = 0.86\%$$

In Panama, factor accumulation causes
 income to grow by 0.86% annually.

This leaves the following residual growth being caused by productivity growth:

$$\hat{A} = 1.73 - 0.86 = 0.87\%$$

The proportions of total growth which are explained by factor accumulation are

$$\frac{0.45}{0.66} = 68\% \text{ for Argentina}$$

$$0.95/1.82 = 52\% \text{ for Uruguay}$$

$$0.86/1.73 = 50\% \text{ for Panama}$$

Factor accumulation is therefore a much larger contributor to overall growth in Argentina than for the other two countries.

The proportions of total growth which are explained by productivity growth are

32% for Argentina, 48% for Uruguay and 50% for Panama.

Productivity growth is therefore a more important contributor to overall growth in Panama.

7.7] If we were to use "days-in-school" instead of "years-in-school", we would obtain much larger differences in human capital: it would increase for rich countries and decrease for poor ones. Accumulated capital would thus explain a larger share of the income differences, thus leaving a smaller residual for productivity differences.