**SOLUTIONS CHAP 4**

**1.** As seen in chapter one, the formula is

$$g = \left(\frac{X_{t+n}}{X_t}\right)^{\frac{1}{n}} - 1 = \left(\frac{6400000000}{2}\right)^{\frac{1}{100000}} - 1 = 0.000218888 = 0.0218\%.$$

**2.** The graphs accompanying this answer are in the file `solutions_graph_chap4b.pdf`.

**2.a)** The initial equilibrium is at point $A$, where population does not grow. With the new seed variety, each worker can produce more; curve $Z$ shifts to the right at $Z'$. In the short run, output jumps to point $B$, resulting in a higher per capita output. This higher output per capita causes the population to grow, as seen on the lower graphic. In the long run, the new steady-state equilibrium is at point $C$, where consumption per capita is the same as before the introduction of the new seed variety. At $L^{SS'}$, the total population size is however larger than initially.

**2.b)** The initial equilibrium is at point $A$, where population does not grow. Population size suddenly drops by half: the economy jumps to point $B$ suddenly. This increases per capita output because there is more land available per worker. As a result, population starts to grow, as indicated by the lower graph. In the long run, the economy returns to its initial point $A$.

**2.c)** The initial equilibrium is at point $A$, where population does not grow. With the destruction of half of the land, output per capita reduces by half for a given population size (assuming constant returns to land). This displaces curve $Z$ to the left to $Z'$. But population size also decreases by half simultaneously, from $L^{SS}$ à $L^{SS'}$. The economy thus jumps from one steady-state to another one without any transition period. Income per capita is the same at the new steady-state but the population size has reduced by half.

**3.** The initial equilibrium is at point $A$, where population does not grow. The population growth curve suddenly shifts from $V$ to $V'$: at each given income level, people want more kids. Population growth suddenly jumps to point $B$ and becomes positive. As population size increases, income per capita decreases. In the long run, income per capita reaches level $y^{SS'}$, which is lower than initially and population size is larger at $L^{SS'}$.

**5.** The ratio of incomes per capita in the steady-state is given by:

$$\frac{\tilde{y}_i}{\tilde{y}_j} = \frac{A^{\frac{1}{1-\alpha}}\left(\frac{\gamma_i}{\delta+n_i}\right)^{\frac{\alpha}{1-\alpha}}}{A^{\frac{1}{1-\alpha}}\left(\frac{\gamma_j}{\delta+n_j}\right)^{\frac{\alpha}{1-\alpha}}} = \left(\frac{\frac{\gamma_i}{\delta+n_i}}{\frac{\gamma_j}{\delta+n_j}}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\frac{0.20}{0.05+0}}{\frac{0.05}{0.05+0.04}}\right)^{\frac{1/3}{1-1/3}} = 2.683.$$

**6.** The graphs accompanying this answer are in the file `solutions_graph_chap4a.pdf`.

In this problem, the population growth rate is *endogenous*, i.e. it depends on the income per worker. More specifically, the line $(n+\delta)k$ is given by $(n_1+\delta)k$ when income per capita is below $f(\bar{k})$, and given by $(n_2+\delta)k$ when income per capita is above $f(\bar{k})$. Note that with $n_2 < n_1$, we simply represent the fact that population growth decreases with income.

There are two possible steady-state equilibria: one at $k_1^{SS}$ with a low income per capita; the other at $k_2^{SS}$ with a high income per capita. This is another instance of a *development trap*: A country that starts off poor has a higher population growth and therefore stays poor because of the *capital dilution* effect. A country that start off rich stays rich because of its lower population growth. In order to sustainably improve the standards of living in the poor country, we would need to find a way to make its capital stock jump above the threshold $\bar{k}$ for a little while with outside development aid for instance.

**7.** We saw that all else equal, the higher the population growth rate, the lower the income per capita in the steady-state. Hence, at the steady-state, income per capita in country $A$ is lower than in country $B$. Because both countries have presently the same income per capita level, this indicates that country $B$ is "farther away" from its steady-state than country $A$. For this reason, income per capita in country $B$ is expected to grow faster in country $B$ than $A$.

This conclusion has intuitive appeal. Indeed, the only thing that differentiates the two countries is the fact that country $A$ has a larger population growth than $B$. It is thus not surprising to see that $B$ will have a larger income growth than $A$ since the burden of the population growth is less severe for $B$ than $A$.

**9)** The graphs accompanying this answer are in the file `solutions_graph_chap4a.pdf`.

**9.a)** See graph.

**9.b)** According to the problem's data, we have:

$$y = \frac{Y}{L} = \frac{L^{\frac{1}{2}}X^{\frac{1}{2}}}{L} = \frac{X^{\frac{1}{2}}}{L^{\frac{1}{2}}}.$$

This implies

$$L = \frac{X}{y^2} = \frac{1000000}{y^2}.$$

This relation appears on graph 7.b).

**9.c)** At the steady-state, the growth rate of population must be zero, i.e. $\hat{L} = 0$. Hence $y = 100$. Substituting the value of $y$ found above, we have:

$$y = \frac{X^{\frac{1}{2}}}{L^{\frac{1}{2}}} = 100,$$

or

$$\frac{1000000^{\frac{1}{2}}}{L^{\frac{1}{2}}} = 100.$$

This implies that the population at the steady-state is $L = 100$.