SOLUTIONS CHAP 3 WEIL 3rd ed

2. In the steady state, the growth rate of capital must be zero because investment in capital is exactly offset by depreciation in capital. (Note: there is no population growth here). If we let the investment rate be given by γ, then the investment level is equal to $\gamma y = \gamma k^{\frac{1}{2}}$. If capital depreciates at rate δ, then the steady state capital stock (k^{ss}) is given by the following equality:

$$\gamma k^{ss\frac{1}{2}} = \delta k^{ss}.$$

With $\gamma = 0.5$ and $\delta = 0.05$, we have $k^{ss} = 10^2 = 100$. At 400, the present capital stock thus exceeds the steady-state stock. This means that the stock will go down over time. Indeed, we can verify this with the following:

$$\Delta k = \gamma k^{\frac{1}{2}} - \delta k = 0.5 * (400)^{\frac{1}{2}} - 0.05 * 400 = -10 < 0.$$

At $k_t = 400$, depreciation exceeds investment.

3. An example in biology is that of the deer population on an island. The quantity of deers that can be supported by the island is limited by the food available on it. If there are very few deers, the food is abundant and their population will grow fast, i.e. births numbers exceed deaths numbers. Conversely, if there are very many deers suddenly brought on the island, food availability per deer will be low and deaths numbers will exceed births; population size goes down. Between these two extremes, there must be a long-run equilibrium number of deers that can be supported indefinitely into the future as the numbers of deaths and births are equal. This is another instance of a steady-state equilibrium in a dynamic setting.

4. Assuming that output per capita can be represented by a Cobb-Douglas functional form, i.e. $y = Ak^{\alpha}$, we have, in the steady-state:

$$\gamma A k^{\alpha} = \delta k.$$

Which yields the following steady-state capital stock:

$$k^{ss} = \left(\frac{\gamma A}{\delta}\right)^{\frac{1}{1-\alpha}}.$$

Inserting this value in the output function, we get the following SS:

$$y^{ss} = Ak^{ss\alpha} = A^{\frac{1}{1-\alpha}}\left(\frac{\gamma}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

If two countries differ solely by their investment rate:

$$\frac{y_i^{ss}}{y_j^{ss}} = \frac{A^{\frac{1}{1-\alpha}}\left(\frac{\gamma_i}{\delta}\right)^{\frac{\alpha}{1-\alpha}}}{A^{\frac{1}{1-\alpha}}\left(\frac{\gamma_j}{\delta}\right)^{\frac{\alpha}{1-\alpha}}} = \left(\frac{\gamma_i}{\gamma_j}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.05}{0.2}\right)^{\frac{1/3}{1-1/3}} = 0.5.$$

In the long run, i.e. at the steady-state, income per capita in country j will be twice that of country i because the latter's savings rate is four times lower.

But if $\alpha = 2/3$, we have

$$\frac{y_i^{ss}}{y_j^{ss}} == \left(\frac{\gamma_i}{\gamma_j}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.05}{0.2}\right)^{\frac{2/3}{1-2/3}} = 0.0625 = \frac{1}{16}.$$

In the long run, income per capita in country j will now be sixteen times that of country i because the latter's savings rate is four times lower.

5.a) (NB Numbers below are taken from the 2nd edition of the book.) If we follow the same procedure as that of the preceding problem, the Solow model predicts that:

$$\frac{y_T^{ss}}{y_B^{ss}} = \left(\frac{\gamma_T}{\gamma_B}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.303}{0.099}\right)^{\frac{1/3}{1-1/3}} = 1.75.$$

In reality, the income per capita ratios is:

$$\frac{14260}{6912} = 2.06,$$

which is somewhat close to the Solow model prediction.

5.b) In this case, the Solow model predicts:

$$\frac{y_N^{ss}}{y_T^{ss}} = \left(\frac{\gamma_N}{\gamma_T}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.075}{0.146}\right)^{\frac{1/3}{1-1/3}} = 0.717.$$

While in reality, the income per capita ratios is:

$$\frac{3648}{17491} = 0.209,$$

which is quite far from the Solow model's predictions.

5.c) In this case, the Solow model predicts:

$$\frac{y_J^{ss}}{y_N^{ss}} = \left(\frac{\gamma_J}{\gamma_N}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{0.313}{0.207}\right)^{\frac{1/3}{1-1/3}} = 1.23.$$

While in reality, the income per capita ratios is:

$$\frac{48389}{43360} = 1.116,$$

which is somewhat close to the Solow model's predictions.

6. The fact that output per capita grows in country X suggests that its capital stock is now below its SS value, and conversely for country Y. According to the Solow model, income per capita and capital per capita at the SS both increase with the savings rate. The fact that both countries now have the same income per capita suggests that the investment rate in country X is higher than in country Y.

7.a) The per capita level of capital (k^{ss}) in SS must respect: $\gamma k^{ss1/2} = \delta k^{ss}$. Hence $k^{ss} = \left(\frac{\gamma}{\delta}\right)^2 = 25$ and $y^{SS} = 25^{\frac{1}{2}} = 5$.

7.b) In period 2, you should get: $k = 16.2$, $y = 4.02$, $\gamma y = 1.005$, $\delta k = 0.81$, $\Delta k = 0.195$. Hence, the period 3 capital stock is $k = 16.395$. And so on. In period 8, you should get: $k = 17.33$, $y = 4.16$, $\gamma y = 1.041$, $\delta k = 0.87$, $\Delta k = 0.174$.

7.c) The growth rate between years 1 and 2 is:

$$g = \frac{X_2 - X_1}{X_1} = \frac{4.02 - 4}{4} = 0.005 = 0.5\%.$$

7.d) The growth rate between years 7 and 8 is:

$$g = \frac{4.16 - 4.14}{4.14} = 0.0048 = 0.48\%.$$

7.e) The growth rate goes down the closer is the economy to its steady-state value.