NAME AND ID: J. PINKMAN

II. PROBLEM

Answer within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

1. Technological progress and economic growth (40 points)
Suppose that the national output of an economy is given by the following function:

$$Y = AK^{\alpha}L^{1-\alpha} = K^{\alpha}(eL)^{1-\alpha}$$

where the variables are as defined in class and $e = A^{\frac{1}{1-\alpha}}$. (The time subscripts have been removed for clarity.) The total investment and depreciation levels are given by $I = \gamma Y$ and $D = \delta K$ respectively, with $\gamma \in (0,1)$ and $\delta \in (0,1)$. Population (L) and total factor productivity (A) grow at constant rates n and $\hat{A}$ respectively. Let eL denote the total quantity of "effective workers" available in this economy.

a) (15) Derive an expression for the steady-state output per effective worker.

We denote variables in terms of "effective workers":

$y_e \equiv \frac{Y}{eL}$, $k_e \equiv \frac{K}{eL}$, $i_e \equiv \frac{I}{eL}$, $d_e \equiv \frac{D}{eL}$

Since $\Delta K = I - D$, we have $\Delta k_e = i_e - d_e$

$\Rightarrow \Delta k_e = \gamma k_e^{\alpha} - (\delta + n + \hat{e}) k_e$

In steady-state, we have $\Delta k_e = 0 \Rightarrow \gamma k_e^{\alpha} = (\delta + n + \hat{e}) k_e$

$\Rightarrow k_e^{ss} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^{\frac{1}{1-\alpha}}$ $\Rightarrow y_e^{ss} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^{\frac{\alpha}{1-\alpha}}$

(Note that $y_e = \frac{Y}{eL} = k_e^{\alpha}$.)

$y_e^{ss} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^{\frac{\alpha}{1-\alpha}}$ is the output per effective worker in steady-state.

b) (10) Derive the rate at which output per worker is growing in the steady-state.

Output per worker $= y = \frac{Y}{L}$

$\Rightarrow y_e \equiv \frac{Y}{eL} = \frac{y}{e} \Rightarrow \hat{y}_e = \hat{y} - \hat{e}$

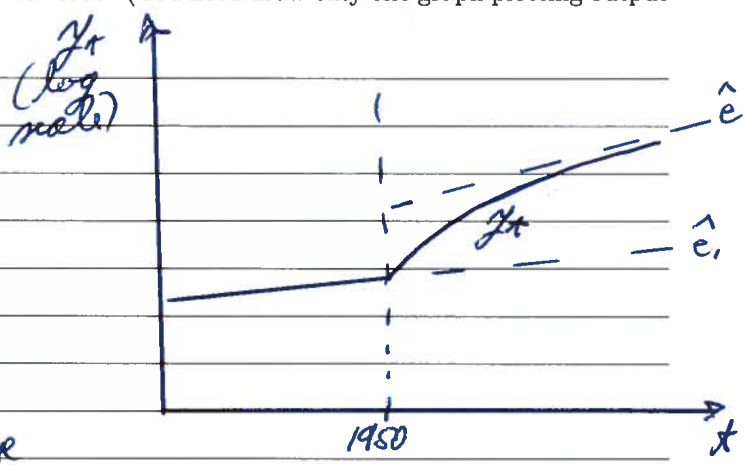
Since $\hat{y}_e^{ss} = 0$, we have $\hat{y}^{ss} = \hat{e}$

Now $e = A^{\frac{1}{1-\alpha}} \Rightarrow \hat{e} = \frac{1}{1-\alpha}\hat{A}$

Therefore: $\hat{y}^{ss} = \frac{1}{1-\alpha}\hat{A}$ is the growth rate of output per worker in steady-state.

c) (15 points) Suppose that in 1950, productivity growth jumps from $\hat{e}_1$ to $\hat{e}_2$, with $\hat{e}_1 < \hat{e}_2$. Assuming that the economy was in steady-state before 1950, show and explain graphically how the output per worker will change over time after 1950. (You need draw only one graph plotting output per worker against time.)

Before 1950, assume output per worker is growing at the steady-state value $\hat{e}_1$. This is represented by the straight line before 1950.



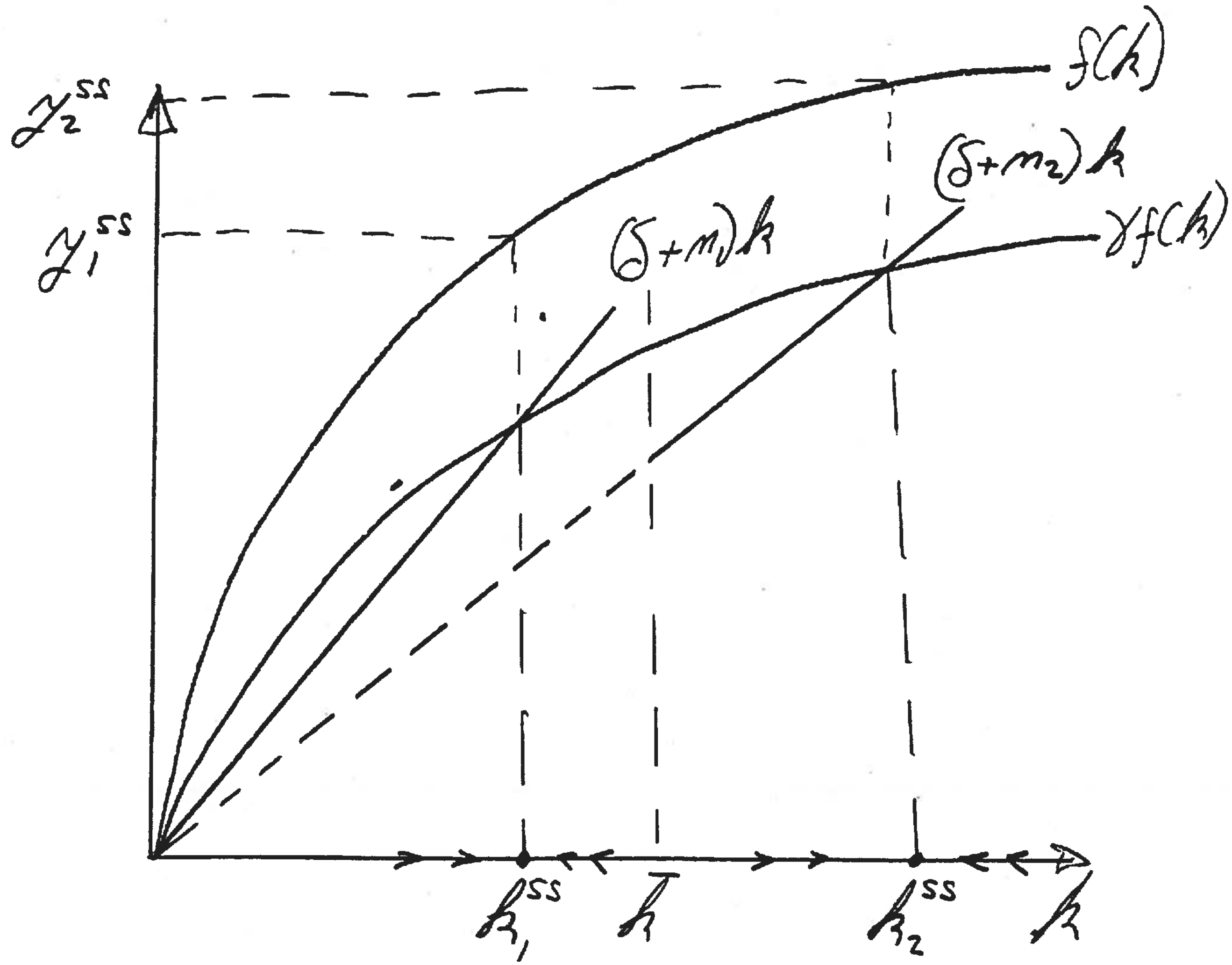
After 1950, the income per worker is converging towards a steady-state with higher growth rate $\hat{e}_2$. This is represented by the dashed line $\hat{e}_2$ with steeper slope. At $t = 1950$, output per worker begins to grow faster to eventually grow at rate $\hat{e}_2$.

2. (30 points) Population growth and economic growth

Consider the Solow model with population growth, as studied in class. Assume that population can grow at two different rates: n_1 and n_2, where $n_1 > n_2$. The population growth rate depends on the level of output per capita (and therefore the level of capital per capita). Specifically, population grows at (high) rate n_1 when $k < \bar{k}$ and at (low) rate n_2 when $k \geq \bar{k}$. We assume that $(n_1 + \delta)\bar{k} > \gamma f(\bar{k})$ and $(n_2 + \delta)\bar{k} < \gamma f(\bar{k})$. Using a graphical analysis, explain why this model leads to bleak predictions regarding the problem of high population growth in poor countries.

Answer: (See accompanying graphic.) In this problem, the population growth rate is *endogenous*, i.e. it depends on the income per worker. More specifically, the line $(n + \delta)k$ is given by $(n_1 + \delta)k$ when income per capita is below $f(\bar{k})$, and given by $(n_2 + \delta)k$ when income per capita is above $f(\bar{k})$. Note that with $n_2 < n_1$, we simply represent the fact that population growth decreases with income.

There are two possible steady-state equilibria: one at k_1^{SS} with a low income per capita; the other at k_2^{SS} with a high income per capita. This is another instance of a *development trap*: A country that starts off poor has a higher population growth and therefore stays poor because of the *capital dilution* effect. A country that starts off rich stays rich because of its lower population growth. In order to sustainably improve the standards of living in the poor country, we would need to find a way to make its capital stock jump above the threshold $\bar{k}$ for a while, say with outside development aid or foreign investment.



y_2^{ss}
y_1^{ss}
$f(k)$
$(\delta+n_2)k$
$(\delta+n_1)k$
$\gamma f(k)$
k_1^{ss}
$\bar{k}$
k_2^{ss}
k