M. Jagger

NAME AND ID:

II. PROBLEM

Answer within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

1. Productivity versus factor accumulation (30 points) Suppose that the national output level of a country is given by the following expression $Y = AK^{\alpha}(hL)^{1-\alpha}$, where the variables are as defined in class and $\alpha = 1/3$. The following table provides the values of each variable in per worker terms for the year 2010.

Country	Year	y	k	h
X	2010	1200	25	45
Z	2010	600	50	15

a) Development accounting (15 points) With the help of the development accounting method, explain why country X is twice as rich as country Z in 2010 by comparing the relative contributions of productivities and factors of production. Explain your steps briefly but clearly.

We have $y = Ak^{\alpha}h^{1-\alpha}$

Let $k^{\alpha}h^{1-\alpha}$ denote the accumulated factors composite index. Therefore,

$$\frac{A_X}{A_Z} = \frac{y_X/y_Z}{k_X^{\alpha}h_X^{1-\alpha}/k_Z^{\alpha}h_Z^{1-\alpha}} = \frac{1200/600}{25^{1/3}45^{2/3}/50^{1/3}15^{2/3}} = \frac{2}{1.65} = 1.21$$

Country X is twice richer than country Z because its productivity is 21% higher, and its larger stock of production factors allocate it to produce 65% more outputs. Accumulated factors are therefore about 3 times more important than productivities in explaining income differences.

b) Growth accounting (15 points) We also have the following data for the same countries X and Z for the year 1960. For each country, calculate the average yearly growth rates of income per worker, physical capital and human capital stocks per worker in the 50 years between 1960 and 2010. Calculate the average yearly productivity growths for each country and use your results to compare the determinants of economic growth in the two countries.

Country	Year	y	k	h
X	1960	100	2	3
Z	1960	50	4	1

$y_{2010} = (1+\hat{y})^{50} y_{1960} \Rightarrow \hat{y} = \left(\frac{y_{2010}}{y_{1960}}\right)^{1/50} - 1 =$ average yearly growth rate

$\Rightarrow \hat{y}_X = \left(\frac{2200}{100}\right)^{1/50} - 1 = 5.1\%$; $\hat{y}_Z = \left(\frac{600}{50}\right)^{1/50} - 1 = 5.1\%$.

Similarly, we have

$\hat{k}_X = \left(\frac{25}{2}\right)^{1/50} - 1 = 5.18\%$; $\hat{k}_Z = \left(\frac{50}{4}\right)^{1/50} - 1 = 5.18\%$

$\hat{h}_X = \left(\frac{45}{3}\right)^{1/50} - 1 = 5.57\%$; $\hat{h}_Z = \left(\frac{15}{1}\right)^{1/50} - 1 = 5.57\%$

Productivity growth is measured as a residual and given by:

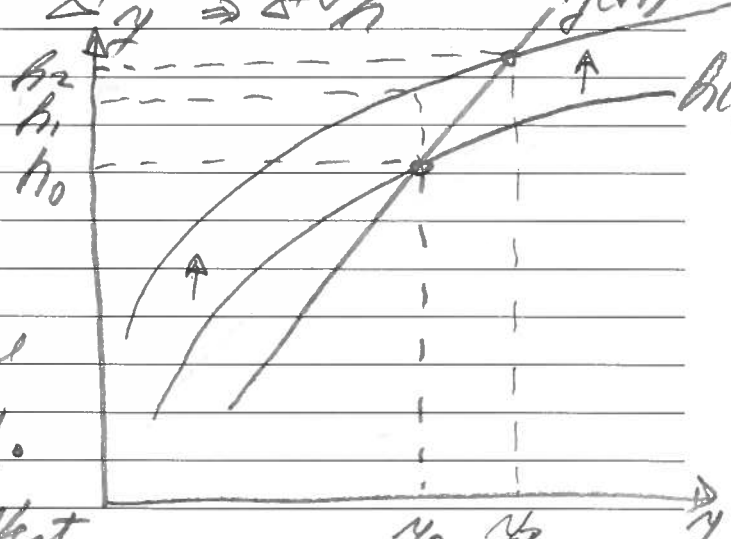
$\hat{A} = \hat{y} - \alpha\hat{k} - (1-\alpha)\hat{h} = 5.1\% - \frac{1}{3}5.18\% - \frac{2}{3}5.57\%$

$= -0.336\%$.

We note that both countries had the same growth rates of output, factors and productivities. The positive growth of output is entirely due to factor accumulation since productivity growth is negative.

2. Health as human capital (10 points) Assume that there is a two-way causality that links health to income. Suppose further that the discovery of a new vaccine allows people to attain higher health levels for given income levels. With the help of a graphic, analyse the effect of the vaccine's discovery on equilibrium income and health levels. Explain briefly but clearly.

We assume that healthier workers can produce more output and that higher income allows workers to be in better health, i.e., $y(h)$ with $\Delta^+ h \Rightarrow \Delta^+ y$ and $h(y)$ with $\Delta^+ y \Rightarrow \Delta^+ h$



Initial income and health are at y_0 and h_0.

The new vaccine shifts curve $h(y)$ upward.

The "direct" effect is to increase health to h_1. But this induces workers to be more productive, raise their income, and increase their health further, i.e., there is a multiplier effect which leads to final health and income levels h_2 and y_2.