2nd mid-term W2013

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II. PROBLEM

You must answer the following questions within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

Growth accounting (30 points) As seen in class, suppose that output per worker is given by the following expression: $y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$, where $\alpha = 1/3$ and subscript t denotes the year. We have the following data for the 25 years between 1975 and 2000: $y_{1975} = 1000$, $y_{2000} = 5000$, $k_{1975} = 5000$, $k_{2000} = 15000$, $h_{1975} = 1000$, $h_{2000} = 2000$.

(1a) (20 points) Calculate the share of total per capita income growth which can be attributed to productivity growth. Begin by writing down the equation which links total per capita income growth to productivity growth and factor growth. (No need to derive it. Just write it down.)

$$\hat{y} = \hat{A} + \alpha \hat{k} + (1-\alpha)\hat{h}$$

In order to estimate $\hat{A}$, we need to calculate $\hat{y}$, $\hat{k}$ and $\hat{h}$. We have:

$$y_{2000} = (1+\hat{y})^{25} \, y_{1975}$$

$$\Rightarrow \hat{y} = \left(\frac{y_{2000}}{y_{1975}}\right)^{1/25} - 1 = 5^{1/25} - 1 = 6.65\%$$

And similarly: $\hat{k} = 3^{1/25} - 1 = 4.49\%$

$\hat{h} = 2^{1/25} - 1 = 2.81\%$

$$\Rightarrow 6.65\% = \hat{A} + \frac{1}{3}(4.49\%) + \frac{2}{3}(2.81\%)$$

$$\boxed{\hat{A} = 3.28\%}$$

$\Rightarrow \frac{3.28}{6.65} = 49.3\%$ is the share of total growth attributed to TFP growth.

(1b) (10 points) In 1957, Robert Solow was ignoring the role of human capital accumulation in explaining output growth. Discuss how this omission would have affected your result in (a).

If we ignore the role of $\hat{h}$, then we have

$\hat{A} = 6.65\% - \frac{1}{3}(4.49\%) = 5.15\%$

NOT REQUIRED

$\Rightarrow \frac{5.15}{6.65} = 77.5\%$ is the share of total growth now attributed to TFP growth.

We therefore obtain that ignoring the role of human capital leads us to over-estimate the role of TFP growth in explaining economic growth.

2. Population growth in the Solow model (30 points) Suppose that there are two countries, B and C, that differ in both their rates of investment and their population growth rates. GDP in country i is given by the following Cobb-Douglas function: $Y_i = AK_i^{\alpha}L_i^{1-\alpha}$, $i \in \{B, C\}$, $\alpha = 1/3$. In country B, investment is 20% of GDP and the population grows at 0% per year. In country C, investment is 5% of GDP and the population grows at 4% per year. The two countries have the same level of productivity, A. In both countries, the rate of depreciation, δ, is 5%.

(2a) (15 points) Based on the Solow model, <u>derive</u> the general expression for the steady state output per capita. Briefly but clearly explain each step in your derivation.

Output per capita is given by

$$y = \frac{AK^{\alpha}L^{1-\alpha}}{L^{\alpha}L^{1-\alpha}} = Ak^{\alpha}$$

The change in capital per worker is

$$\Delta k = \gamma A k^{\alpha} - (\delta + n)k$$

where γk^{α} is the investment level, δk is the depreciation of capital and nk is the capital dilution effect due to population growth.

In the steady-state, we have

$$\Delta k = 0 \Rightarrow \gamma A k^{\alpha} = (\delta + n)k \Rightarrow k^{ss} = \left(\frac{A\gamma}{\delta + n}\right)^{\frac{1}{1-\alpha}}$$

The steady-output per worker is thus:

$$y^{ss} = A(k^{ss})^{\alpha} = A^{\frac{1}{1-\alpha}}\left(\frac{\gamma}{\delta + n}\right)^{\frac{\alpha}{1-\alpha}}.$$

(2b) (15 points) Calculate the ratio of steady-state output per capita between the countries and discuss the effect of population growth on income. Discuss and justify how a more realistic value of α would affect this result.

We have $\dfrac{y_B}{y_C} = \dfrac{A^{\frac{1}{1-\alpha}}\left(\frac{\gamma_B}{\delta+n_B}\right)^{\frac{\alpha}{1-\alpha}}}{A^{\frac{1}{1-\alpha}}\left(\frac{\gamma_C}{\delta+n_C}\right)^{\frac{\alpha}{1-\alpha}}}$

With $\alpha = 1/3$, we have $\frac{\alpha}{1-\alpha} = 1/2$ and

$$\frac{y_B}{y_C} = \left(\frac{\gamma_B}{\gamma_C} \cdot \frac{\delta+n_C}{\delta+n_B}\right)^{1/2} = \left(\frac{.20}{.05} \cdot \frac{.05+.04}{.05+0}\right)^{1/2}$$

$$= 2.7$$

Country B's per capita income is 2.7 times higher than country C because of both higher investment and lower population growth. Population growth alone accounts for a factor of $\left(\frac{.05+.04}{.05+0}\right)^{1/2} = 1.34$ due to the capital dilution effect. A more realistic value for α is $2/3$ when one includes human capital as an accumulated factor. This leads to an income ratio of $\left(\frac{.20}{.05} \cdot \frac{.05+.04}{.05+0}\right)^{2} = 51.8$

when one includes both larger investment and popul. growth differences.

EITHER IS GOOD.

If one only looks at popul. growth differences, then the long-run income ratio prediction is $\left(\frac{.05+.04}{.05}\right)^{2} = 3.24$.