BEYONCEY (1)

NAME AND ID:

II. PROBLEM

You must answer the following questions within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

1. The Solow model (50 points)
Assume that an economy can be represented by the following per capita output function: $y = Ak^{\alpha}$, where $\alpha = 1/3$ et $A = 5$. (Each variable is defined as seen in class.) The depreciation rate for (physical) capital is given by $\delta = 10\%$ and the investment rate is given by $\gamma = 20\%$.

a)(10 points) Suppose that at period t, the capital stock per capita is $k_t = 10$. Compare the per capita income at period t with the long run, steady-state equilibrium per capita income as predicted by the Solow model. Show briefly how the steady-state equilibrium is derived. (No graphic here.)

At time t, we have $y_t = Ak_t^{\alpha} = 5(10)^{1/3} = 10.77$

In the L.R., we have $i = d$, i.e., investment is equal to depreciation. We have $i = \gamma y$ and $d = \delta k$. Hence, in the L.R., S.S. equilibrium, we have:

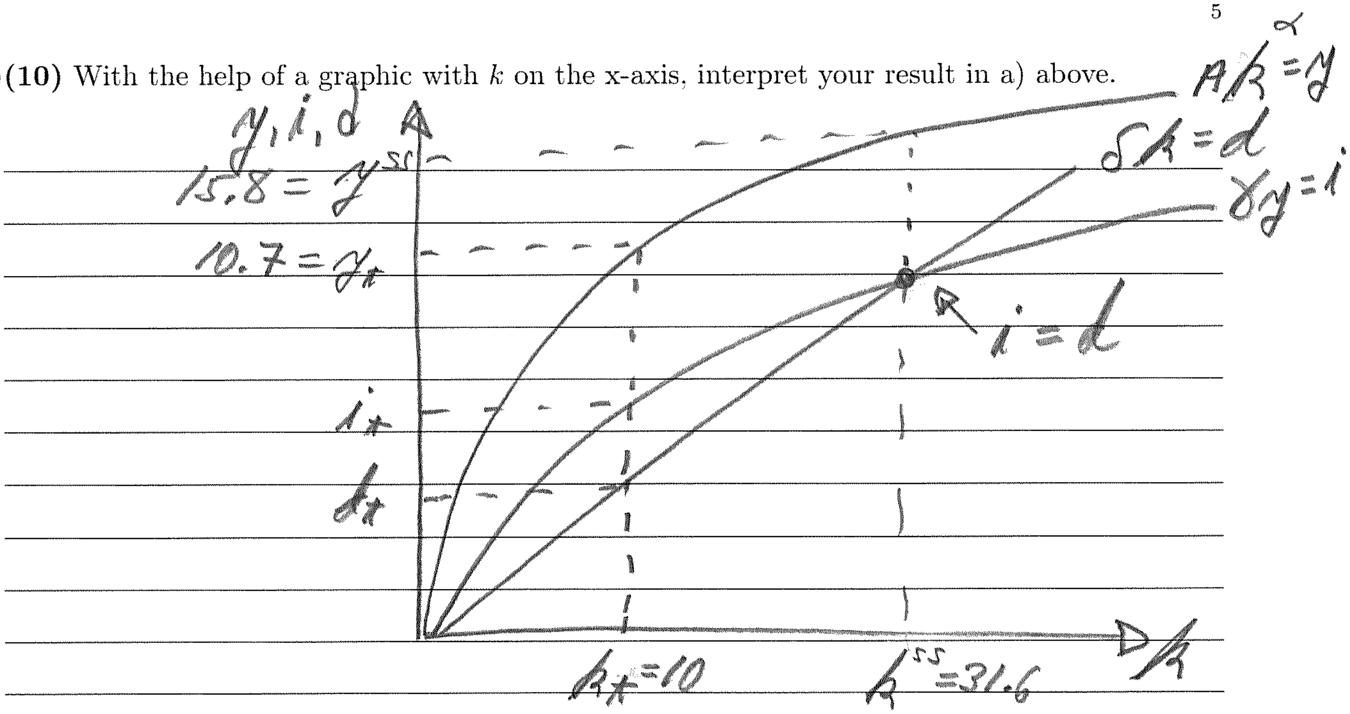
$$\gamma A k^{\alpha} = \delta k \Rightarrow k^{ss} = \left(\frac{\gamma A}{\delta}\right)^{\frac{1}{1-\alpha}}$$

$$\Rightarrow k^{ss} = (0.2 \cdot 5/0.1)^{3/2} = 31.6$$

The L.R. income per capita is thus $y^{ss} = A(k^{ss})^{1/3} = 5(31.6)^{1/3} = 15.8$

We thus have $y_t < y^{ss}$.

b)(10) With the help of a graphic with k on the x-axis, interpret your result in a) above.



at $k_t = 10$, we have $i_t > d_t$, such that the capital stock is increasing. This increase continues until $k = 31.6$, at which point we have $i = d$, such that investment only replaces depreciated capital. We therefore have a steady-state.

c) (10) Assuming still that $k_t = 10$, calculate the per capita income growth rate during period t, i.e., $\Delta y_t / y_t$ where $\Delta y_t = y_{t+1} - y_t$. (Explain briefly your mathematical steps.)

at $k_t = 10$, we have $i_t = \gamma y_t = 0.2 \cdot 10.77 = 2.15$

and also: $d_t = \delta k_t = 0.1 \cdot 10 = 1$

Hence: $k_{t+1} = k_t + i_t - d_t = 10 + 2.15 - 1 = 11.15$

Consequently: $y_{t+1} = A(k_{t+1})^\alpha = 5(11.15)^{1/3} = 11.2$

The growth rate of income is thus:

$$\frac{\Delta y_t}{y_t} = \frac{y_{t+1} - y_t}{y_t} = \frac{11.2 - 10.77}{10.77} = 3.99\%$$

d) (10) Suppose instead that at period t, the capital stock per capita is $k_t = 20$. Recalculate the corresponding growth rate of income per capita during period t. (Just show the calculations this time.) Compare this result with the one obtained in c) above and explain the difference. (Be as complete as possible but no more.)

$$k_t = 20 \Rightarrow y_t = 5(20)^{1/3} = 13.57$$

$$\Rightarrow i_t = \gamma y_t = 0.2 \cdot 13.57 = 2.714$$

$$d_t = \delta k_t = 0.1 \cdot 20 = 2$$

$$\Rightarrow k_{t+1} = k_t + i_t - d_t = 20 + 2.714 - 2 = 20.714$$

$$\Rightarrow y_{t+1} = 5(20.714)^{1/3} = 13.73$$

$$\Rightarrow \frac{\Delta y_t}{y_t} = \frac{13.73 - 13.57}{13.57} = 1.19\%$$

The growth rate is now lower than in c) because with a larger capital stock, the economy is closer to its steady-state. This is due to the presence of decreasing marginal product of capital (decreasing MPK).

e) (10) The per capita GDP growth rate in China between 1990 and 2010 was about 10% on average. In the past five years or so, it has been consistently declining to reach 6.3% in 2017. Explain how the Solow model can help us predict this slowing down of the per capita GDP growth rate. Discuss by invoking the concept of "convergence".

The Solow model predicts that countries that are farther away from their steady-state in terms of capital stock will growth faster. But as the capital stock increases and becomes closer to its steady-state value, the growth rate goes down. We say that the economy "converges" towards its steady-state.

The reduction in the growth rate in China is consistent with this convergence prediction in the Solow model.