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II. PROBLEM

You must answer the following questions within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

1. A theory of intertemporal choice (35 points) Suppose that Penelope lives two periods only, $t \in \{1, 2\}$. Y_{dt} is her disposable income at period t and W_1 is her initial wealth at period 1. She can save or borrow at interest rate r and cannot leave a bequest or unpaid debt after period 2. C_t is her consumption level at period t.

a) (15 points) Let S_1 represent the savings level in period 1. Write down the two separate equations representing consumption levels at period 1 and period 2 respectively. Combine these two equations in order to represent the intertemporal budget constraint and show that it can be interpreted as an equality between the present discounted value of consumption levels and the present discounted value of available resources. Explain each mathematical step in words.

[A] $C_1 = Y_{d1} + W_1 - S_1$: Consumption at period 1 is equal to all available resources at $t=1$ minus savings.

[B] $C_2 = (1+r) S_1 + Y_{d2}$: Consumption at $t=2$ equals its disposable income plus savings augmented by the interest rate.

We have from [A]: $S_1 = Y_{d1} + W_1 - C_1$

Insert into [B] to get:

$C_2 = (1+r)(Y_{d1} + W_1 - C_1) + Y_{d2}$

Rearranging gives:

[C] $C_1 + \dfrac{C_2}{1+r} = Y_{d1} + W_1 + \dfrac{Y_{d2}}{1+r}$

The left-hand side of [C] is the present value of consumption while the R.H.S. is the P.V. of available resources.

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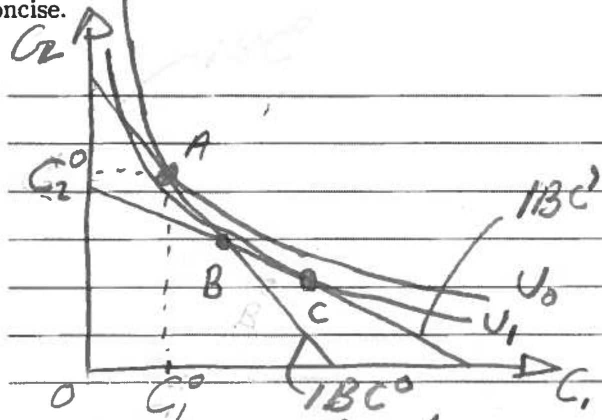
b)(10 points) With the help of a graphical analysis, concoct an example in which a decrease in the rate of interest leads Penelope to go from a net saver to a net borrower. (Assume convex indifference curves.) Interpret your results as completely as possible while remaining clear and concise.



Suppose point B corresponds to no savings, i.e. $S_1 = 0$. At point A, we thus have $S_1 > 0$, i.e. net positive on the intertemporal budget constraint IBC^0.

A drop in r leads to a new IBC with gentler slope that passes thru point B, such as IBC^1.

It is then possible to have a new tangent indifference curve that yields consumption point C, i.e. to the right of point B. This corresponds to negative savings. This increase in C_1 is due to the lower opportunity cost of period 1 consumption when r decreases.

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c) **(10 points)** Take the following values: $Y_{1d} = 30,000\$$, $Y_{2d} = 40,000\$$, $W_1 = 10,000\$$ and $r = 10\%$. Suppose that Penelope wants to have a consumption level equal to $35,000\$$ in period 2. How much should she save? Explain your steps.

$C_2 = (1+r)(Y_{d1} + W_1 - C_1) + Y_{d2}$

$\Rightarrow 35 = (1.1)(30 + 10 - C_1) + 40$

Using the IBC, we set $C_2 = 35000$ and then find the consumption at $t=1$ that corresponds. Solving this gives

$C_1 = 44545$

We thus have the following savings at $t=1$:

$S_1 = Y_{d1} + W_1 - C_1 = 30 + 10 - 44{,}545 = -4545$ #

The consumer is a net borrower at $t=1$ since he consumes less than the disposable income at $t=2$.

2. The Solow model (35 points)

a) **(20 points)** A country is described by the Solow model, with a production function of $y = k^{1/3}$. Suppose that today, k is equal to 600. The fraction of output invested is 30% and the depreciation rate is 2%. How does the output per worker today compare with the steady-state one? Explain your conclusion.

Investment is $i = \gamma k^{1/3} = 0.3 \cdot (600)^{1/3} = 2.53$

Depreciation is $d = \delta k = 0.02 \cdot 600 = 12$

The total change in capital is thus: $\Delta k = i - d = 2.53 - 12 = -9.47 < 0$

Since the stock of capital is

decreasing, this means that output per worker today is larger than the S-S one.

b) (15 points) Suppose that the national production function for the Canadian economy can be expressed as $Y = AK^{\alpha}L^{1-\alpha}$, where each variable is as described in class. Explain how one could estimate the value of parameter α for Canada. Be as complete as possible while remaining clear and concise.

In a competitive market for capital, its return must be equal to its marginal product, i.e. $r = MPK \Rightarrow r = \alpha A K^{\alpha-1} L^{1-\alpha}$. This implies

$$\frac{rK}{Y} = \frac{\alpha A K^{\alpha} L^{1-\alpha}}{A K^{\alpha} L^{1-\alpha}} = \alpha$$

Since rK = total revenues from capital in the economy, α can be estimated by calculating the ratio of aggregate capital income over GDP. Such numbers are easily found in people's income tax filings.