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NAME AND ID:

II. PROBLEM

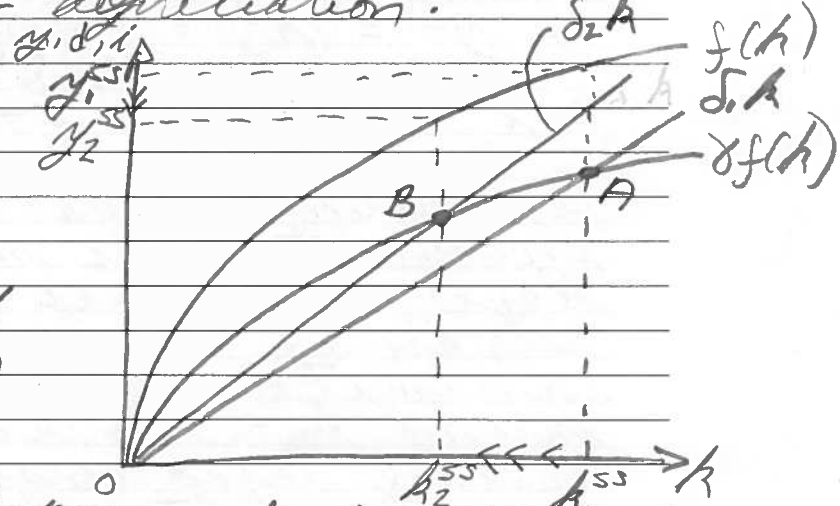
You must answer the following questions within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

1. The Solow model and climate change (30 points)

A country's economy is represented by the basic Solow model, with a production function of $y = f(k)$, where k is capital per worker and there is a diminishing marginal product of capital. The investment rate (γ) is constant throughout. Suppose that because of more severe weather conditions, climate change increases the depreciation rate of capital. We want to analyze the possible effect on the economy with the help of the Solow model.

1.a) (20) Assume that the effect of climate change takes the form of a sudden increase in the depreciation rate of capital, say from δ_1 to δ_2, with $\delta_2 > \delta_1$, which occurs at year 2000. With the help of graphical analysis, discuss the short-run and long-run effects of this sudden increase in the depreciation rate. Use two graphical representations: one that depicts output, investment and depreciation as functions of capital; another that depicts output as a function of time. Explain the graphics and interpret your results.

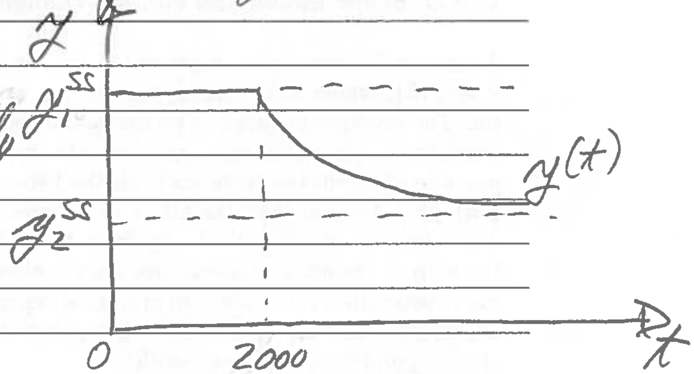
The change in capital stock is given by
$\Delta k = \gamma f(k) - \delta k$.
The long-run steady state is therefore given by $\gamma f(k) = \delta k$, i.e., investment = depreciation.



Suppose that at year 2000, the economy is at the S-S point A with depreciation rate δ_1.

An increase in the depreciation rate from δ_1 to δ_2 in year 2000 means that the depreciation of capital exceeds investment in the short run, i.e., $f(k_1)$, k_1. The stock of capital consequently decreases until it reaches a new, long-run S-S at point B. The corresponding trajectory for output is illustrated below.

Output per worker decreases fast initially and converges towards y_2^{ss}.



1.b) (10) Assume now that output is represented by the Cobb-Douglas function $y = k^{\alpha}$. Derive the expression for the steady-state consumption level (c^{ss}) as function of the investment and depreciation rates. (Make sure to show all the steps with a short explanation.) With the help of a graph that plots c^{ss} as a function of γ, show how the curve changes when the depreciation rate increases from δ_1 to δ_2.

We have $\gamma (k^{ss})^{\alpha} = \delta k$ $\quad k^{ss} = \left(\gamma/\delta\right)^{\frac{1}{1-\alpha}}$

$\Rightarrow y^{ss} = (k^{ss})^{\alpha} = \left(\frac{\gamma}{\delta}\right)^{1 \cdot \alpha}$

$\Rightarrow c^{ss} = (1-\gamma) y^{ss} = (1-\gamma)\left(\gamma/\delta\right)^{\frac{\alpha}{1-\alpha}}$

Consumption is equal to the part of income that is not invested. Hence, an increase in the investment rate has an ambiguous effect on consumption since it increases income but decreases the share of income being consumed. This is illustrated by the following graph.

We can see from the above equation that a larger δ leads to a lower c^{ss}.

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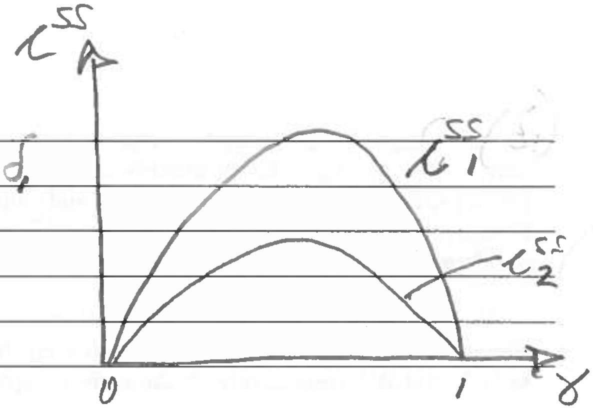
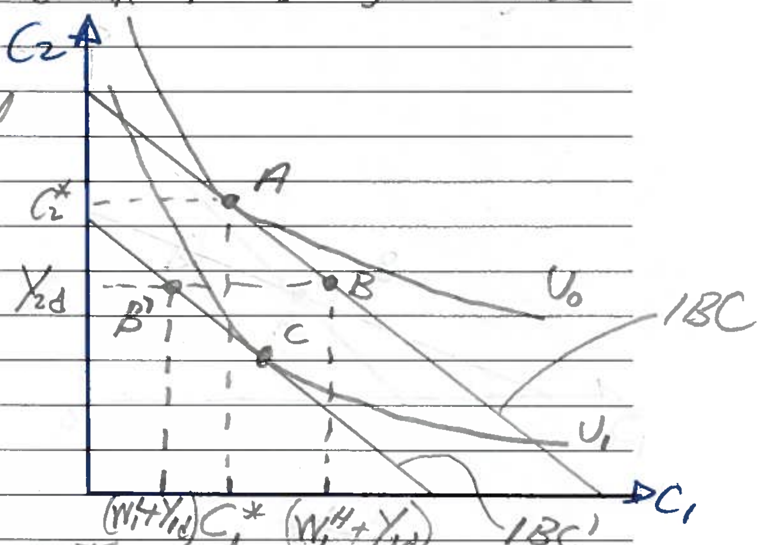
Consequently,
$c_2^{ss} \not> c_1^{ss}$ when $\delta_2 > \delta_1$
for any given γ,
as illustrated.



2. A theory of intertemporal choice (20 points) Suppose that Ronaldo lives for two periods only, $t \in \{1, 2\}$. Y_{dt} is his disposable income at period t and W_1 is his initial wealth at period 1. He can save or borrow at interest rate r and cannot leave a bequest or unpaid debt after period 2. C_t is his consumption level at period t and S_1 represents the savings level in period 1. Ronaldo's indifference curves between the two period's consumption levels are convex.

With the help of graphical analysis, illustrate a case in which a drop in the initial wealth leads Ronaldo to go from being a net saver to a net borrower. Denote the high and low initial wealths as W_1^H and W_1^L respectively. Make sure to explain your steps clearly, both graphical and mathematical.

We have $C_1 = W_1^H + Y_{1d} - S_1$ and $C_2 = Y_{2d} + (1+r)S_1$. This gives $C_2 = (1+r)(W_1^H + Y_{1d} - C_1) + Y_{2d}$, as represented by curve IBC.



With W_1^H, Ronald consumes C_1^* in period 1 at A and is thus a net saver since $C_1^* < W_1^H + Y_{1d}$.

Suppose a drop in W_1 shifts the intertemporal budget constraint down from IBC to IBC'. The no-savings, no borrowing point goes from B to B'. Point C denotes the consumption levels on IBC' where consumption at period 1 exceeds $W_1^L + Y_{1d}$. Ronaldo is thus now a net borrower.