NAME AND ID: L. H.

II. PROBLEM

You must answer the following questions strictly within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

The Solow model (60 points) Suppose that output per worker is given by $y_t = k_t^\alpha$ and that investment and depreciation rates are respectively given by γ and δ. (Recall that the marginal product of capital is equal to $\alpha k_t^{\alpha-1}$.)

a) **(25 points)** Suppose that you wanted to test the validity of the Solow model's predictions. To this end, your first task is to estimate the value of α. Explain how you would proceed with a complete mathematical demonstration of the data required and some underlying assumptions.

Given that $Y_t = K_t^\alpha L_t^{1-\alpha}$, we can show that $\alpha = \frac{rK_t}{Y_t}$, i.e., α is simply the share of total income that comes from capital earnings. Indeed, assuming a competitive capital rental market implies

$r = MPK = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$.

$$\Rightarrow \frac{rK_t}{Y_t} = \frac{\alpha K_t^{\alpha-1} L_t^{1-\alpha} K_t}{K_t^\alpha L_t^{1-\alpha}} = \alpha$$

In order to estimate α, one therefore only needs to obtain data about GDP values and the aggregate value of capital earnings.

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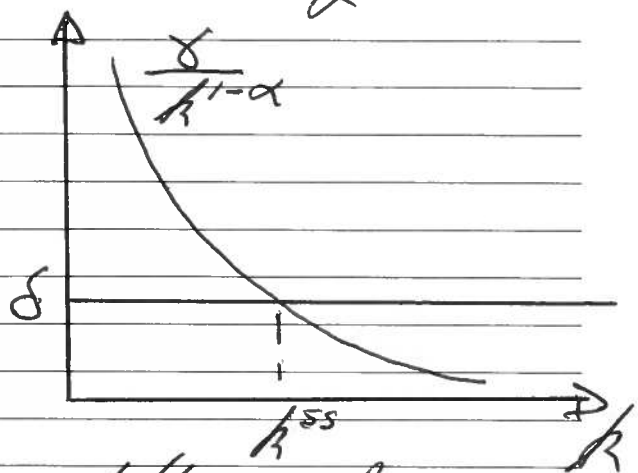
b) (25 points) With the help of a graphic, explain why the growth rate of capital per worker is larger the farther away it is from its steady state value.

The growth of capital per worker is given by

$$\frac{\Delta k}{k} = \frac{\gamma k^{\alpha} - \delta k}{k} = \frac{\gamma}{k^{1-\alpha}} - \delta$$

The steady-state capital value is therefore obtained by

$$\frac{\gamma}{k^{1-\alpha}} = \delta .$$



The graph shows that $\Delta k/k$ increases as k_t moves away from k^{ss} (in absolute values) or in particular, $\Delta k/k$ is largest when k_t is very small.

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c) (10 points) Given your result in part b) above, comment on the following assertion by a famous expert in world affairs: "Given the much larger growth rates that China has experienced over the past 30 years, it is safe to say that the standard of living of the Chinese citizens will far exceed that of the USA citizens by the end of the century."

The result in part b) suggests that one cannot assume that the high past growth rates will continue forever into the future. As the stock of capital becomes closer to its steady-state value, the growth rate will slow down. Based on the experiences of Japan or South Korea, it is safer to assume that the standard of living in China will converge towards that of the U.S.A. instead of overtaking it.