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II. PROBLEM

You must answer the following questions strictly within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

The Solow model (60 points) Suppose that at any period t, the aggregate output of an economy (Y_t) depends on the total amounts of workers (L_t) and (physical) capital (K_t) only. This is represented by function F as follows: $Y = F(K, L)$, where subscripts t are removed for simplicity.

a) (10 points) Propose a property for function F which allows us to say that the output per worker (y) depends only on the amount of capital per worker (k), that is, $y = f(k)$. Demonstrate why.
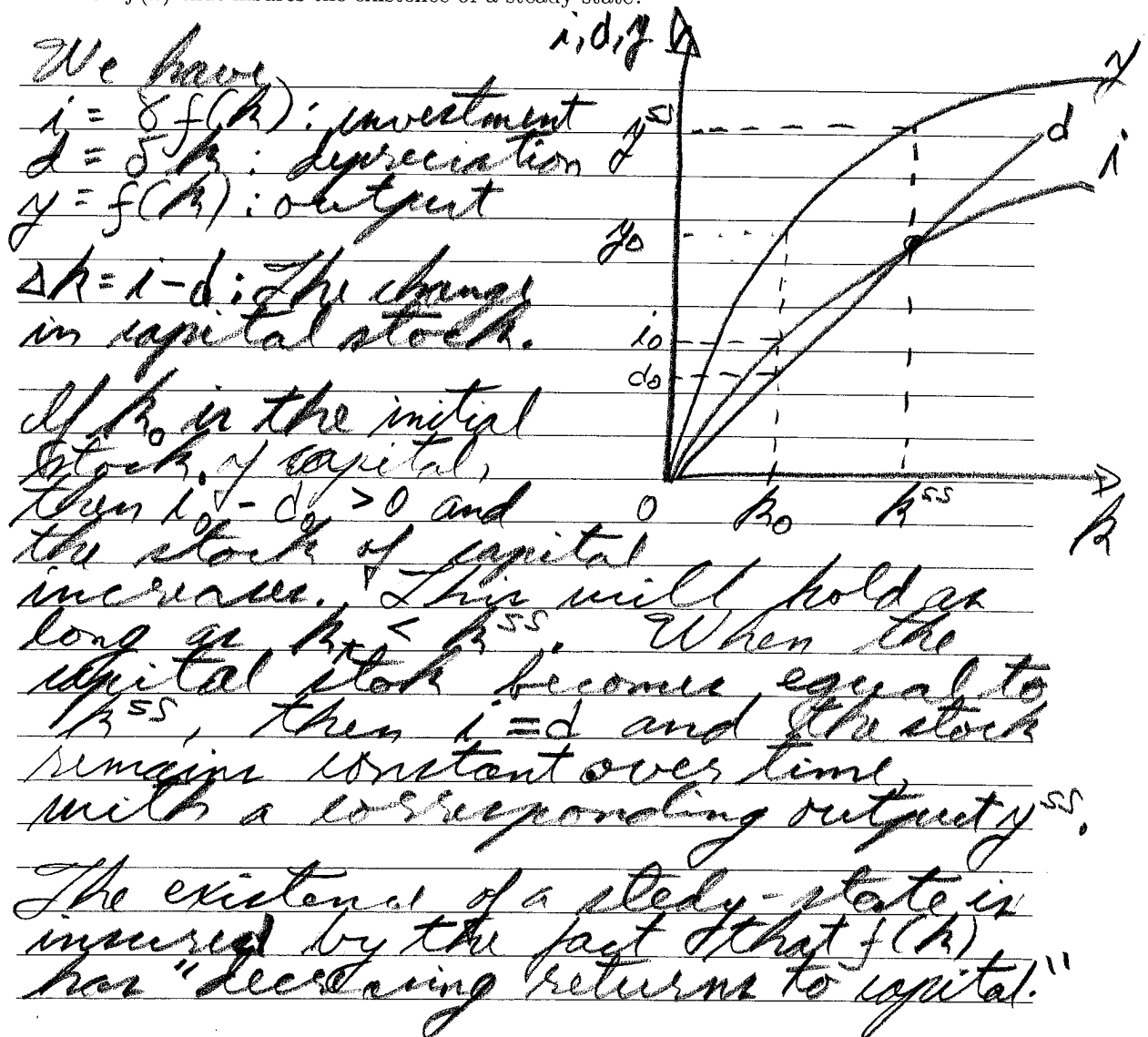
The property is "constant returns to scale", that is, $zY = F(zK, zL)$, for any value of z.

Let $z = \frac{1}{L}$. Then $\frac{Y}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right) = F(k, 1)$

We can therefore say that $y = f(k)$.

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b) (25 points) Suppose that at every period, workers invest a constant proportion $\gamma \in (0,1)$ of their income into increasing the capital stock but that the capital stock depreciates linearly at constant rate $\delta \in (0,1)$. With the help of a graphic, describes the mechanism through which the economy will reach a steady-state in the long run. What is the assumption that must be imposed on function $f(k)$ that insures the existence of a steady state?

We have,

$i = \delta f(k)$: investment

$d = \delta k$: depreciation

$y = f(k)$: output

$\Delta k = i - d$: The change in capital stock.

If k_0 is the initial stock of capital, then $i_0 - d_0 > 0$ and the stock of capital increases. This will hold as long as $k_0 < k^{ss}$. When the capital stock becomes equal to k^{ss}, then $i = d$ and the stock remains constant over time, with a corresponding output y^{ss}.

The existence of a steady-state is insured by the fact that $f(k)$ has "decreasing returns to capital."

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c) (**25 points**) Suppose that up to time $\bar{t}$, the economy was operating at its long-run steady state values corresponding to an investment rate γ_1. At $\bar{t}$, the saving rate suddenly jumps to $\gamma_2 > \gamma_1$. Draw a graphic describing the evolution of income per worker over time and discuss the effects on the growth rate.

At $\bar{t}$, the invest-ment rate suddenly jumps above the depreciation rate. This causes the capital stock to begin increasing, along with the output level. Eventually, a new steady state at k_2^{ss} and y_2^{ss} will be reached.

Note that due to decreasing returns to capital, income grows rapidly at first but slows down as it gets nearer to the steady-state value.