

**3. (40 points) Trade and Investment in the National Accounts** The following table provides flows and stocks in the national accounts of a fictitious economy. The various variables are as defined in the text on *Trade and Investment in the National Accounts*. ( $Y_t$  = GDP;  $Y_t^N$  = GNP;  $CA_t$  = current account balance;  $Q_t$  = imports;  $B_t$  = net foreign asset holdings)

The GDP function is given by  $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$ , where  $\alpha = 1/3$ . (Recall that the marginal product of capital is then given by  $MPK = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}$ .) The depreciation rate on physical capital is  $\delta = 10\%$ . Human capital plays no role in this economy.

The stock variables denote values at the *beginning* of each period. To simplify, the total population size is constant and equal to one (i.e.,  $L_t = 1$ , for all  $t$ ); hence, the numbers in the table can also be interpreted as *per capita* values. **In year 2000, the economy is closed with respect to the rest of the world.**

year ( $t$ )	$Y_t$	$C_t$	$I_t$	$G_t$	$X_t$	$Q_t$	$NX_t$	$B_t$	$rB_t$	$Y_t^N$	$CA_t$	$K_t$	$r_t$
2000	100		20	25	0			0			0	250	
<b>SS closed</b>				25	0			0			0		
<b>open</b>				25	15			-107.9			0		12%

  

year ( $t$ )	$Y_t$	$C_t$	$I_t$	$G_t$	$X_t$	$Q_t$	$NX_t$	$B_t$	$r_t B_t$	$Y_t^N$	$CA_t$	$K_t$	$r_t$
2000	100	<b>55</b>	20	25	0	<b>0</b>	<b>0</b>	0	<b>0</b>	<b>100</b>	0	250	<b>13.3%</b>
<b>SS closed</b>	<b>89.5</b>	<b>46.6</b>	<b>17.9</b>	25	0	<b>0</b>	<b>0</b>	0	<b>0</b>	<b>89.5</b>	0	<b>179</b>	<b>16.7%</b>
<b>open</b>	<b>105.5</b>	<b>49.04</b>	<b>29.3</b>	25	15	<b>12.84</b>	2.16	-107.9	<b>-12.95</b>	<b>92.56</b>	0	<b>293</b>	12%

(A) (10) By reproducing the table above in your booklet, fill in the blanks regarding year 2000 only for now. (NB You must write explicitly the equations that you use. Each time an equation is used for the first time, provide a brief description in words.) Calculate: i) the total factor productivity ( $A_{2000}$ ); ii) the rental cost of capital ( $r_{2000}$ ); iii) the (pre-tax) labor income ( $w_{2000}$ ).

$Y = C + I + G$ : IN A CLOSED ECONOMY, THE DEMAND FOR OUTPUT  $Y$  IS COMPOSED OF CONSUMPTION, INVESTMENT AND GOVERNMENT PURCHASES. HENCE,  $100 = C + 20 + 25$  IMPLIES  $C_{2000} = 55$ . IN A CLOSED ECONOMY, IMPORTS ARE NIL, HENCE,  $Q_{2000} = 0$ .

$NX = X - Q$ : NET EXPORTS IDENTITY. HERE,  $NX = 0 - 0 = 0$  IN A CLOSED ECONOMY.

$B = 0$  HENCE  $rB = 0$ : IN A CLOSED ECONOMY, THERE IS NO FOREIGN ASSET AND THUS THE INVESTMENT INCOME BALANCE IS NIL (OR NO DEBT SERVICE).

$Y^N = Y + rB$ : THE GROSS NATIONAL PRODUCT IS GDP PLUS NET INCOME FROM FOREIGN ASSETS. HERE, GDP=GNP SINCE THERE IS NO FOREIGN ASSET.

$CA = B_{t+1} - B_t = rB_t + NX_t$ : THE CURRENT ACCOUNT BALANCE GIVES THE CHANGE IN NET FOREIGN ASSETS. HERE,  $CA_{2000} = 0$  BECAUSE THE ECONOMY IS CLOSED.

WITH  $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$  AND  $L = 1$ , WE HAVE  $100 = A_{2000} (250)^{0.333}$ , SO THAT TFP IS  $A_{2000} = 15.88$ .

IN A COMPETITIVE ECONOMY, THE RENTAL COST OF CAPITAL IS EQUAL TO ITS MARGINAL PRODUCT:  $r = \alpha AK^{\alpha-1} = 0.333(15.88)(250)^{-0.666} = 13.34\%$ .

TOTAL LABOR INCOME IS 2/3 OF GDP AND IS EQUAL TO THE WAGE RATE SINCE  $L = 1$ . HENCE,  $w = 0.666Y = 66.67$ . (ALTERNATIVELY, THE WAGE IS EQUAL TO THE MARGINAL PRODUCT OF LABOUR:  $w = (1 - \alpha)AK^{\alpha}L^{-\alpha} = 0.666AK^{\alpha}$  (SINCE  $L = 1$ ) AND THUS  $w = 0.666Y = 66.67$ .)

- (B) (10) The economy is still closed. Assume that the investment rate ( $I/Y = 0.2$ ) does not change over time and that the government does not invest, i.e.,  $G_t$  denotes only consumption by the government. According to the basic Solow model, what will be the steady-state (long run) capital stock and per capita income level. Use these results to fill in the blanks in line **SS closed** in the table. Assume throughout that  $G_t = 25$ . Calculate the labor wage.

IN SS, INVESTMENT EQUALS DEPRECIATION  $\gamma Y = \delta K$ . HENCE,  $\gamma AK^{\alpha} = \delta K$  WHERE  $\gamma = 0.2$ . THIS IMPLIES  $K^{ss} = (0.2 \cdot 15.88/0.1)^{3/2} = 179$ . AND THUS  $Y^{ss} = 15.88 \cdot (179)^{1/3} = 89.5$ ,  $I = 0.2Y = 17.9$ ,  $C = Y - I - G = 89.5 - 17.9 - 25 = 46.6$ ,  $Y^N = Y = 89.5$ .

$$r = MPK = \alpha AK^{\alpha-1} = 0.333 \cdot 15.88(179)^{-2/3} = 16.67\%$$

$$w = MPL = (2/3)Y = 59.7$$

- (C) (10) Suppose now that the economy is open to the rest of the world and that **capital is perfectly mobile**. The return to capital in the rest of the world is constant and equal to  $r_W = 12\%$ . (Recall that “return to capital” and “rental cost of capital” are synonymous.) Calculate the corresponding stock of capital, GDP and labor wage. How do labor wages compare between the open and the closed economy? Explain.

PERFECT CAPITAL MOBILITY IMPLIES  $r = r_W$ . HENCE,  $r = 12\%$ . WITH  $MPK = r$ , WE HAVE  $K = (r_W/(\alpha \cdot A))^{1/(\alpha-1)} = (0.12/(0.333 \cdot 15.88))^{-3/2} = 293$ . HENCE,  $Y = 15.88(293)^{1/3} = 105.5$  AND  $w = (2/3)Y = 70.3$ .

IN THE CLOSED ECONOMY, WE HAD  $w = 59.7$ . HENCE, WAGES HAVE INCREASED IN THE OPEN ECONOMY. THIS IS BECAUSE ECONOMIC OPENNESS HAS *attracted more capital* INTO THE ECONOMY WHICH CONTRIBUTES TO *raise the marginal product of labor*.

- (D) (10) In the open economy of part (C) above, let  $B_t$  denote the net foreign asset holdings. Assume that these assets take the form of physical capital only; there is thus no government bonds. Fill in the blanks in the table. (Take into account the fact that the economy is in steady-state and that  $CA = 0$ .) In steady-state, are people better off with the open economy or with the closed economy? Explain.

IN SS, INVESTMENT IS EQUAL TO DEPRECIATION. HENCE,  $I = \delta K = 0.1 \cdot 293 = 29.3$  (NOTE, HOWEVER, THAT INVESTMENT IS NOW COMPOSED OF BOTH DOMESTIC AND FOREIGN INVESTMENT, SO THAT THERE IS A DISCONNECT BETWEEN DOMESTIC SAVINGS AND INVESTMENT, I.E.,  $I \neq \gamma Y^N$ .) WE HAVE  $rB = 0.12 \cdot (-107.9) = -12.94$  AND  $Y^N = Y + rB = 105.5 - 12.94 = 92.56$ .

IN SS, WE HAVE  $B_{t+1} = B_t$ . *The rest of the problem was trickier because of the presence of depreciation in  $B_t$ . In class, we assumed that government bonds did not depreciate. This cannot be assumed with physical capital and we had not seen in class how to incorporate the depreciation of foreign assets into the national accounts. For this reason, this part (D) will count as bonus points only, i.e., the exam will count on 160 points instead of 170 and 10 extra points can be accumulated here.*

WITH DEPRECIATION OF PHYSICAL CAPITAL, WE HAVE  $B_{t+1} = B_t + rB_t + NX_t - \delta B_t$ . IN SS,  $B_{t+1} = B_t$  THIS IMPLIES  $NX_t = (\delta - r)B_t = (0.1 - 0.12)(-107.9) = 2.16$ . HENCE,  $Q = X - NX = 15 - 2.16 = 12.84$  AND  $C = Y - I - G - NX = 49.04$ . IN THE CLOSED ECONOMY, WE HAD  $C = 46.6$ . HENCE, PRIVATE CONSUMPTION INCREASED WITH OPENNESS WHILE GOVERNMENT EXPENDITURES ARE THE SAME. THIS SUGGESTS THAT PEOPLE ARE BETTER OFF UNDER OPENNESS. EVEN THOUGH THE COUNTRY IS NOW INDEBTED TOWARDS FOREIGN INVESTORS, THE INCREASED LABOUR PRODUCTIVITY STEMMING FROM THE CAPITAL INFLUX MORE THAN COMPENSATES FOR THIS LIABILITY.