II. PROBLEMS

1. (40 points) Capital mobility and economic growth

Assume that the output per capita of a country is given by $y = Ak^\alpha$. (Note that this implies that the marginal product of capital is equal to $\alpha A k^{\alpha-1}$.)

a) **(10)** Assume that the country is CLOSED to the rest of the world such that its capital is NOT mobile. Assuming a savings rate of γ and a capital depreciation rate of δ, derive the long-run output per capita. How does it depend on the savings rate? (To be solved with equations. No graphic.)

Answer: The long-run equilibrium is determined by the investment being equal to capital depreciation, that is,

$$\gamma A k^\alpha = \delta k.$$

Rearranging this equation, we get

$$k^{SS} = \left(\frac{\gamma A}{\delta}\right)^{\frac{1}{1-\alpha}}.$$

This leads to a long-run income per capital equal to

$$y^{SS} = A^{\frac{1}{1-\alpha}} \left(\frac{\gamma}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

We therefore see that the long-run output per capita depends positively on the savings rate. This suggests that in order to raise income, people must reduce their consumption.

b) **(10)** Assume now that capital is perfectly mobile with the rest of the world. State clearly and briefly what the law of one price for capital movements says. Show that the equilibrium stock of capital is independent of the country's savings rate. (To be solved with equations. No graphic.) Explain intuitively.

Answer: The law of one price says that with perfect mobility of capital between one country and the rest of the world, the return to capital in the one country must be equal to the one in the rest of the world. If this were not the case, investors would move their capital to where the return is highest, thus re-establishing the equality in capital returns.

In a competitive economy, the rate of return to capital must be equal to its marginal product, that is,

$$r = \alpha A k^{\alpha-1}.$$

If r^W denotes the (given) return to capital on world markets, the law of one price implies that $r = r^W$, and thus

$$r^W = \alpha A k^{\alpha-1}.$$

This implies that the stock of capital in the country is given by

$$k^{SS} = \left(\frac{\alpha A}{r^W}\right)^{\frac{1}{1-\alpha}}.$$

We therefore see that the stock of capital does not depend on the country's savings rate.

c) **(5)** Does your answer to (b) imply that countries that save more are no better off than countries that save nothing when capital is perfectly mobile? Explain.

Answer: No. A country that has a relatively low savings rate may benefit from an influx of capital from the rest of the world as it potentially increases local wages a lot.

However, another part of the additional output will have to be used to repay the foreign owners of capital. So countries with relatively high savings rates will also benefit from additional income coming from their investments in foreign countries.

An alternative explanation can be made by comparing the GDP of a country with its GNP, all in per capita terms. We have

$$GNP_t = GDP_t + rB_t^f,$$

where B^f denotes the country's net holding of foreign assets. A country with a relatively high savings rate will have $B_t^f > 0$, whereas one with a relatively low savings rate will have $B_t^f < 0$. We therefore see that even though two countries may have the same GDP per capita with perfect capital mobility (since their capital stocks are equal), the country with the high savings rate will have a higher GNP due to its interest income receipts from foreign countries.

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We have seen that the current account balance of an open economy was given by the following identity:

$$B_{t+1}^f - B_t^f = rB_t^f + NX_t, \tag{1}$$

where B_t^f denotes net foreign asset holdings, r is the rate of interest on assets, and NX_t is net exports.

d) **(5)** Explain intuitively <u>in words</u> what identity (1) means.

Answer: The variables on the left-hand side (LHS) of the identity denote *stock* values. The difference is the change in net foreign asset holdings between two consecutive years. For example, if the country is a net debtor with respect to the rest of the world – i.e. $B_t^f < 0$ – a positive value for LHS indicates a reduction in its foreign liabilities.

The right-hand side (RHS) variables are *flow* values. They indicate what causes net foreign asset holdings to increase in a given year. For instance, if $B_t^f < 0$, then the country must export more than it imports in to cover interest payments rB_t^f on its net foreign debt.

e) **(10)** Is it always better to have a positive current account balance? Answer from the perspective of Canada's experience over the past 50 years or so.

Answer: Not necessarily. A positive current account balance in Canada means that Canadian savers are investing more in foreign countries than foreigners are investing in Canada. As a result, the stock of capital in Canada is lower than it would be with, say, a zero current account balance. A lower stock of capital leads to lower wages for workers. The advantage with a positive current account balance is that savers may obtain a better return by investing abroad, as well as diversifying the risk.

From the 1960s to the early 2000s, Canada has generally had negative current account balance. However, its foreign debt to GDP ratio has not increased because the additional capital allowed workers to be more productive.

II. PROBLEM

Answer within the space provided. Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

Technological progress and economic growth (60 points)
Suppose that the national output of an economy is given by the following function:

$$Y = AK^{\alpha}L^{1-\alpha} = K^{\alpha}(eL)^{1-\alpha}$$

where the variables are as defined in class and $e = A^{\frac{1}{1-\alpha}}$. (The time subscripts have been removed for clarity.) The total investment and depreciation levels are given by $I = \gamma Y$ and $D = \delta K$ respectively, with $\gamma \in (0,1)$ and $\delta \in (0,1)$. Population (L) and total factor productivity (A) grow at constant rates n and $\hat{A}$ respectively. Let eL denote the total quantity of "effective workers" available in this economy.

a) (25 points) Derive an expression for the steady-state output per effective worker. At what rate is the output per worker growing in this steady-state?

We first define the following variables in terms of "effective workers":

$y_e = \frac{Y}{eL}$, $k_e = \frac{K}{eL}$, $i_e = \frac{I}{eL}$, $d_e = \frac{D}{eL}$

Since $\Delta K = I - D$, we have $\Delta k_e = i_e - d_e$

$\Rightarrow \Delta k_e = \gamma k_e^{\alpha} - (\delta + n + \hat{e}) k_e$

In steady-state: $\Delta k_e = 0 \Rightarrow \gamma k_e^{\alpha} = (\delta + n + \hat{e}) k_e$

$\Rightarrow k_e^{SS} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^{\frac{1}{1-\alpha}} \Rightarrow \boxed{y_e^{SS} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^{\frac{\alpha}{1-\alpha}}}$

$\left(NB\ y_e = \frac{Y}{eL} = k_e^{\alpha}\right)$ This is the SS output per effective worker.

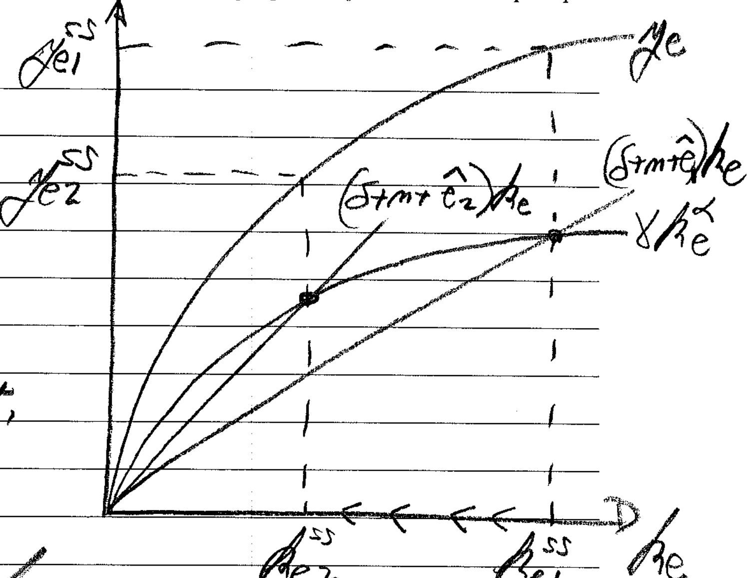
Since output per worker $= y = \frac{Y}{L}$, we have $y_e = \frac{y}{e} \Rightarrow \hat{y_e} = \hat{y} - \hat{e}$. In steady-state, $\hat{y_e} = 0 \Rightarrow \hat{y} = \hat{e}$. Output per worker is growing at rate $\hat{e} = \frac{1}{1-\alpha}\hat{A}$!

b) (10 points) Explain briefly (in words only) how it is possible that output per worker be growing at a faster rate than $\hat{A}$.

We found above that output per worker is growing at rate

$\frac{1}{1-\alpha}\hat{A} > \hat{A}$. This growth rate is larger than the total factor productivity growth rate because as TFP increases, so does the accumulation of capital per worker (k).

c) (25 points) Suppose that in 1950, productivity growth jumps from $\hat{e}_1$ to $\hat{e}_2$, with $\hat{e}_1 < \hat{e}_2$. Assuming that the economy was in steady-state before 1950, show and explain graphically how the output per worker will change over time after 1950.



An increase in $\hat{e}$ moves the economy towards a new steady-state, i.e. from y_{e1}^{ss} to y_{e2}^{ss}.

In the long run, y_{e2}^{ss} being constant, we have $\hat{y}_2 = \hat{e}_2$.

Output per worker will be growing at a faster rate than before 1950.