# ECO 2143 : MACRO II
# SOLUTIONS FINAL EXAM APR. 2008

## PROBLEMS

### 1) Technological progress in the Solow Model

$$Y = AK^{\alpha}L^{1-\alpha} = K^{\alpha}(eL)^{1-\alpha}$$

a) $$\frac{Y}{eL} = \frac{K^{\alpha}}{(eL)^{\alpha}} \cdot \frac{(eL)^{1-\alpha}}{(eL)^{1-\alpha}} \Rightarrow \boxed{y_e = k_e^{\alpha}}$$

b) Since $k_e = \frac{K}{eL}$, we have

$$\Delta k_e = \gamma k_e^{\alpha} - (\delta + n + \hat{e}) k_e .$$

In steady-state, $\Delta k_e = 0$.

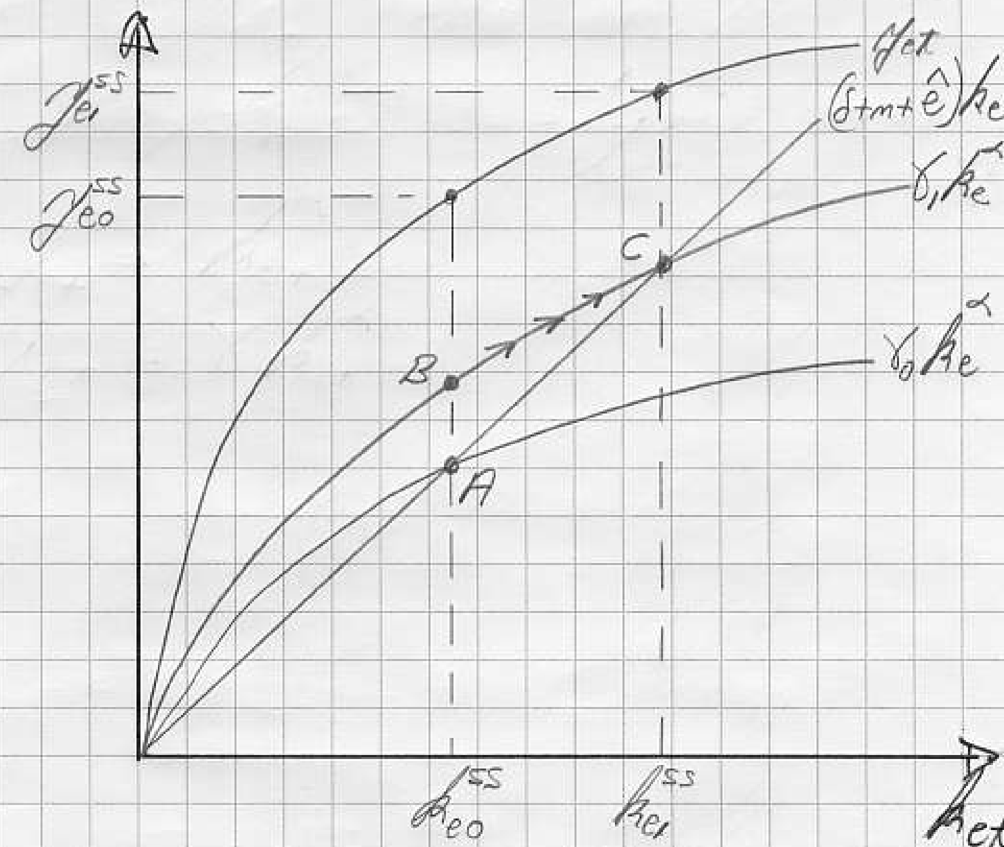
$$\Rightarrow \gamma k_e^{\alpha} = (\delta + n + \hat{e}) k_e$$

$$\Rightarrow k_e^{SS} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^{\frac{1}{1-\alpha}}$$

$$\Rightarrow \boxed{y_e^{SS} = (k_e^{SS})^{\alpha} = \left(\frac{\gamma}{\delta + n + \hat{e}}\right)^{\frac{\alpha}{1-\alpha}}}$$

This is the S-S value for $y_e$.

c)




A jump in the savings rate means that the savings curve jumps up from $\gamma_0 k_{et}^{\alpha}$ to $\gamma_1 k_{et}^{\alpha}$. With an initial S-S at point A, savings suddenly exceeds "effective" depreciation at point B. So $k_e$ increases. In the long run, there is a new S-S equilibrium at point C, with $k_{e1}^{SS} > k_{e0}^{SS}$ and $y_{e1}^{SS} > y_{e0}^{SS}$.

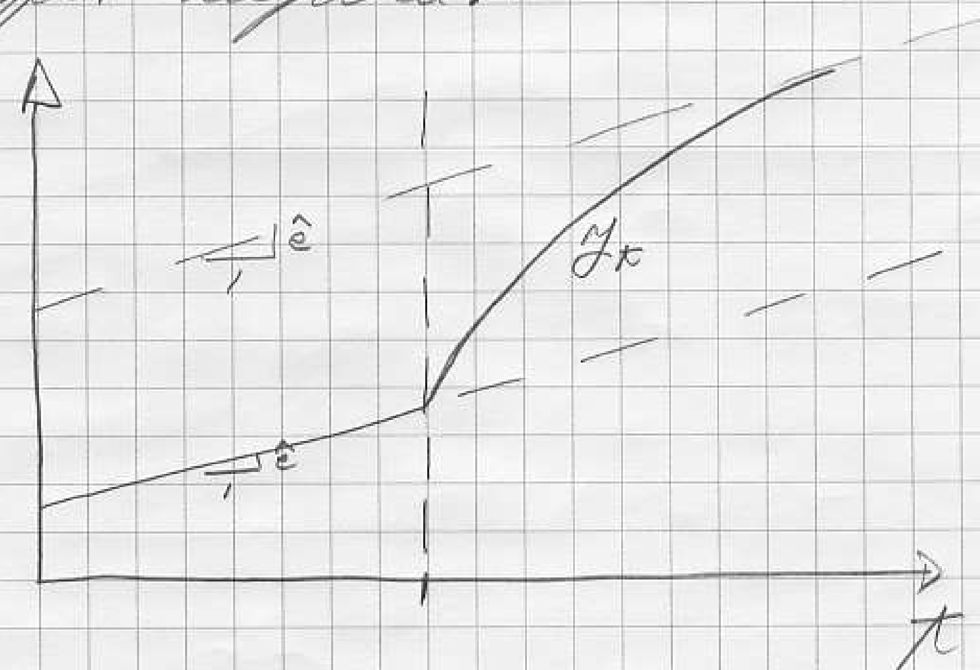
As for per capita income, we have

$$y_{et} = \frac{y_t}{e_t} \Rightarrow y_t = e_t \, y_{et}$$

$$\Rightarrow \hat{y}_t = \hat{y}_{et} + \hat{e}$$ (hats denote growth rates)

In S-S, $\hat{y}_{et} = 0 \Rightarrow \hat{y}^{ss} = \hat{e}$

In S-S, income per capita grows at the same rate as technology. Moreover, since $e_t$ is unchanged, a higher $y_{et}^{ss}$ implies a higher $y_t$. Over time, we have the following evolution of income per capita:



## 2) Trade and investment in the national accounts

a) Future output can increase with a higher capital stock, which results from higher investment:

$$K_{t+1} = K_t - \delta K_t + I_t$$

In a closed economy, we must have:

$$I_t = S_t,$$

i.e. in order to increase investment, people must save more and thus lower their consumption as

$$Y_t = C_t + I_t \Rightarrow I_t = Y_t - C_t$$

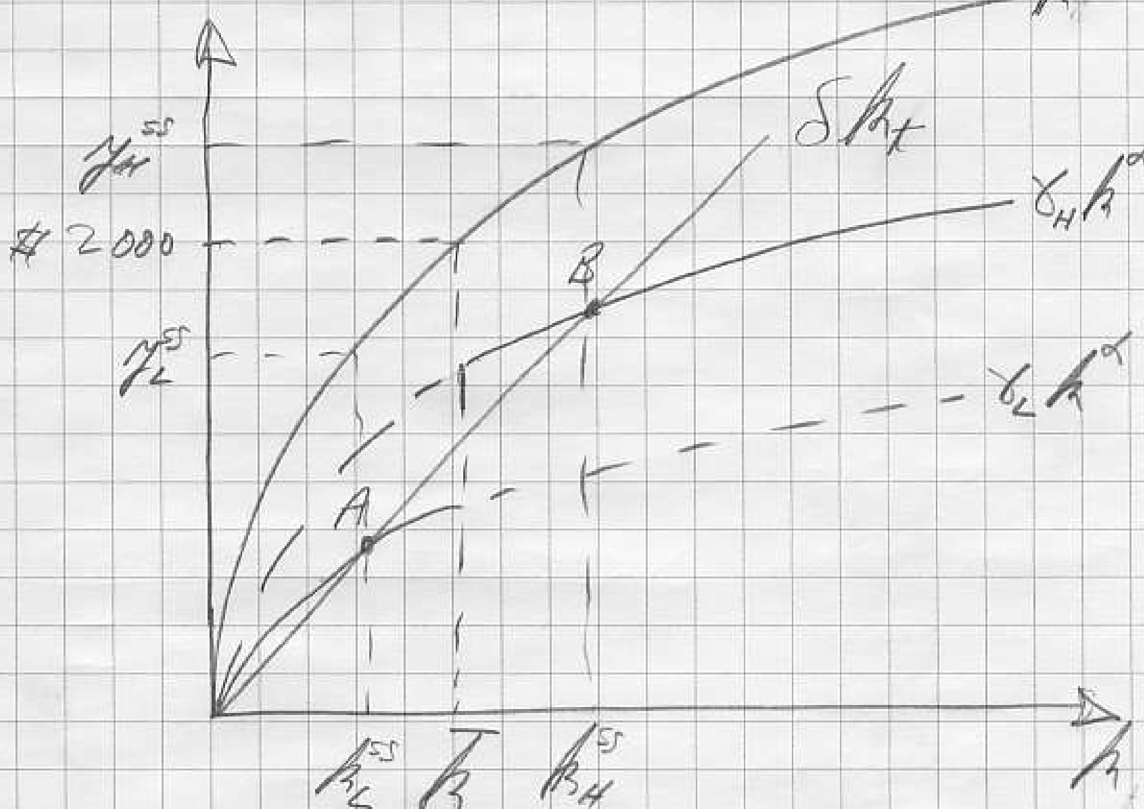
The problem is that very poor people may not be able to save more. Suppose that people with income below \$2000 can only save 5% of their income while those with income above \$2000 can save 10% of their income. We have:

$y_t = k_t^\alpha$ : income per capita

$i_t = \gamma k_t^\alpha$ : investment " "

$d_t = \delta k_t$ : depreciation " "

$\gamma_L = 5\%$
$\gamma_H = 10\%$.




Points A and B represent two stable steady-states. In a closed economy, if a country starts out poor, it stays poor, and conversely. Only outside help can help to push the economy from a low equilibrium A to a high equilibrium B.

## b-) OPEN ECONOMY

In an open economy, savings must not be equal to investments as people can borrow from outside. We have:

$$Y_t + Q_t = C_t + I_t + X_t$$

$$\Rightarrow I_t = Y_t - C_t - NX_t \quad \text{where } NX_t = X_t - Q_t \text{ is net exports}$$

$I_t$ can thus be increased with a lower $NX_t$ <u>without</u> any reduction in consumption ($C_t$). This can potentially allow a country to come out of a development trap. Indeed, when capital is perfectly mobile, the marginal product of capital must be equal to the world rate of interest, i.e.

$$\alpha k_t^{\alpha-1} = r^W$$

$$\Rightarrow k^* = \left(\frac{r^W}{\alpha}\right)^{\frac{1}{\alpha-1}} = \left(\frac{\alpha}{r^W}\right)^{\frac{1}{1-\alpha}}$$

$$\Rightarrow \boxed{y^* = \left(\frac{\alpha}{r^W}\right)^{\frac{\alpha}{1-\alpha}}}$$

With perfect mobility of capital, output per worker is <u>independent</u> of a country's savings rate. This is because foreign investors will invest in a country where the marginal product of capital is high.

(c) If $k_L^{ss} < k^*$, then

$$MPK_L^{ss} = \alpha \left(k_L^{ss}\right)^{\alpha-1} > r^W.$$

By opening up to the rest of the world, foreigners invest into the country. Hence, the country's net foreign asset holdings must decrease, i.e.

$$B_1^f - B_0^f < 0.$$

The current account balance is negative.

We have

$$B_1^f - B_0^f = rB_0^f + NX_0$$

With $B_0^f = 0$, we have

$$B_1^f = NX_0 < 0$$

Through imports, the country is increasing its capital stock. But $B_1^f$ being negative, the country will have to pay back an interest service $rB_1^f$ in the future. Whether this is good or not depends on the benefit of higher worker productivity following those investments.