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II. PROBLEM

Your answers must be accompanied with clear explanations. Graphs and equations without explanations will not get you far.

1.(20 points) Productivity differences You are given the following observations for Canada and South Korea concerning per capita output, physical capital and human capital (all relative to the USA values). You assume that the output per capita is given by the following relation:

$$y = Ak^{\alpha}h^{1-\alpha},$$

where $\alpha = 1/3$ and A denotes total factor productivity (TFP).

	y	k	h
Canada	0.75	0.86	1.01
South Korea	0.54	0.73	0.93

(a) Calculate the ratio of TFP between Canada and South Korea.

$A = y / k^{\alpha}h^{1-\alpha} \Rightarrow \dfrac{A_{SK}}{A_{CAN}} = \dfrac{y_{SK} / k_{SK}^{\alpha} h_{SK}^{1-\alpha}}{y_{CAN} / k_{CAN}^{\alpha} h_{CAN}^{1-\alpha}}$

$\Rightarrow \dfrac{A_{SK}}{A_{CAN}} = \dfrac{\dfrac{0.54}{(0.73)^{1/3}(0.93)^{2/3}}}{\dfrac{0.75}{(0.86)^{1/3}(1.01)^{2/3}}} = \dfrac{0.54/0.86}{0.75/0.96}$

$= 0.804$

Productivity in SK is 80.4% that of Canada, thereby causing SK to have a 19.6% lower income level.

(b) Calculate what is the most important cause of South Korea's lower income: factor accumulation or productivity? Discuss briefly.

$\dfrac{k_{SK}^{\alpha} h_{SK}^{1-\alpha}}{k_{CAN}^{\alpha} h_{CAN}^{1-\alpha}} = \dfrac{(0.73)^{1/3}(0.93)^{2/3}}{(0.86)^{1/3}(1.01)^{2/3}} = \dfrac{0.86}{0.96} = 0.90$

This suggests that lower factor accumulation causes income in S.K. to be 10% lower than Canada's.

Hence, productivity is about twice as important in explaining its lower income level.

2. (25 points) Trade and Investment in the National Accounts

The following table provides flows and stocks in the national accounts of a fictitious economy. The various variables are as defined in the text on *Trade and Investment in the National Accounts.* (Y_t^N = GNP; CA_t = current account balance; Q_t = imports)

The stock variables denote values at the *beginning* of each period. The return on government bonds (foreign or domestic) is $r = 8\%$ and the depreciation rate on domestic capital is $\delta = 10\%$. Government bonds are the only form of foreign asset holdings and they do not depreciate. The population size and productivity level are assumed constant over time and we abstract from the possibility of human capital accumulation. (NO government expenditure.)

year (t)	Y_t	C_t	I_t	X_t	Q_t	NX_t	B_t	rB_t	Y_t^N	CA_t	K_t
2000	100	60	20	30	10	20	0	0	100	20	250
2001	99	50	35	30	16	14	20	1.6	100.6	15.6	245

DESCRIPTION

(a) (15) Fill in the blanks in the table above by providing a brief explanation for each equation being used for the first time.

$Y = C + I + X - Q$: Production (or GDP) must be equal to the domestic use of resources.

$X_{2000} = 100 - 60 - 20 + 10 = 30$

$Q_{2001} = -99 + 50 + 35 + 30 = 16$

$NX = X - Q$: Net exports or "trade balance"

$NX_{2000} = 30 - 10 = 20$ $NX_{2001} = 30 - 16 = 14$

rB is investment income balance

$rB_{2000} = r \cdot 0 = 0$

$Y^N = Y + rB$ is gross national product which accounts for foreign debt payments or foreign investment income.

$Y^N_{2000} = 100 + 0 = 100$

$CA_t = B_{t+1} - B_t = rB_t + NX_t$ is the current account balance. It corresponds to the change in international investment position.

$CA_{2000} = 0 + 20 = 20$

$B_{2001} = B_{2000} + rB_{2000} + NX_{2000} = 0 + 0 + 20 = 20$

$rB_{2001} = 0.08 \cdot 20 = 1.6$

$Y^N_{2001} = 99 + 1.6 = 100.6$

$CA_{2001} = 1.6 + 14 = 15.6$

$K_{t+1} = K_t - \delta K_t + I_t$: The change in the stock of capital is equal to investment minus depreciation.

$K_{2001} = 250 - 0.1 \cdot 250 + 20 = 245$

(b) (10) Explain why there is a drop in GDP between 2000 and 2001 and discuss whether it was a bad thing or not for the economy as a whole.

GDP decreases because of the lower stock of capital, i.e., $K_{2001} < K_{2000}$. This is due to the fact that domestic investments in 2000 were insufficient to make up for capital depreciation.

There was, however, a net accumulation of foreign assets that generated extra income ($rB_{2001} = 1.6$).

As a result, total national income increased from 100 to 100.6.

This suggests that the drop in GDP due to lower domestic investment may not have been a bad thing if compensated by an investment in foreign assets.

3. The Solow model (25 points) Suppose that at any period t, the aggregate output of an economy (Y_t) depends on the total amounts of workers (L_t) and (physical) capital (K_t) only. This is represented by function F as follows: $Y = F(K, L)$, where subscripts t are removed for simplicity. (NB Productivity growth and population growth are zero in this problem.)

a) (5 points) Propose a property for function F which allows us to say that the output per worker (y) depends only on the amount of capital per worker (k), that is, $y = f(k)$. Demonstrate why.

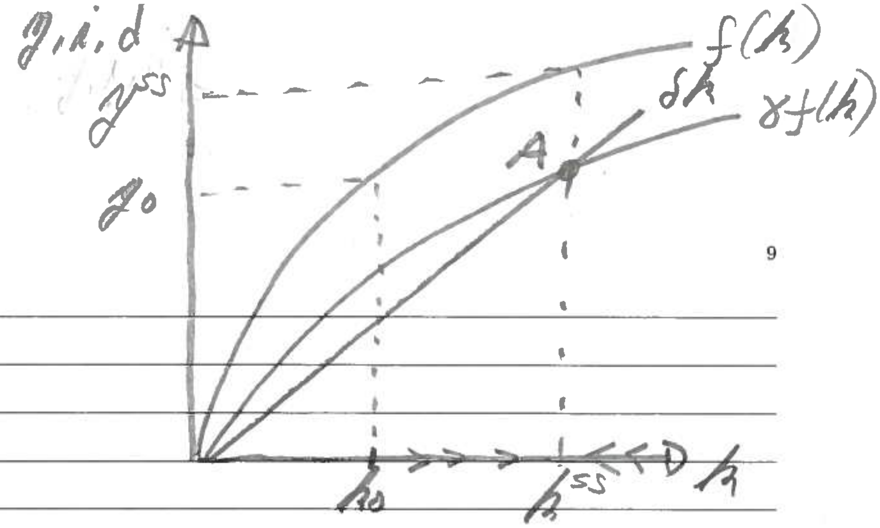
Constant returns to scale (CRS).

$CRS \Rightarrow F(K/L, L/L) = \frac{F(K,L)}{L}$

Hence: $y \equiv \frac{F(K,L)}{L} = F(k,1) = f(k)$

b) (10 points) Suppose that at every period, workers invest a constant proportion $\gamma \in (0,1)$ of their income into increasing the capital stock but that the capital stock depreciates linearly at constant rate $\delta \in (0,1)$. With the help of a graphic, describes the mechanism through which the economy will reach a steady-state in the long run. What is the assumption that must be imposed on function $f(k)$ that insures the existence of a steady state?

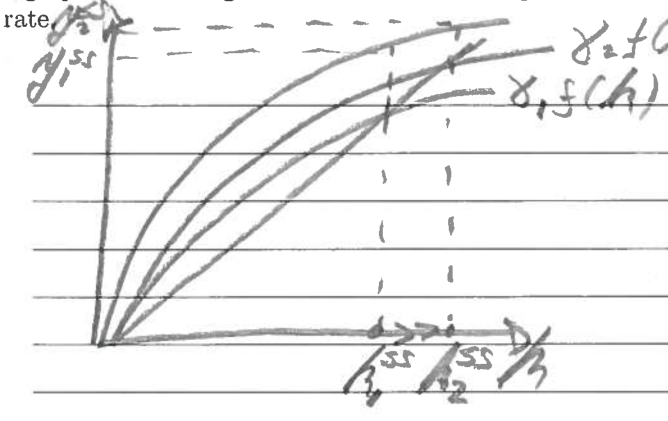
The change in capital stock ($\Delta k_t \equiv k_{t+1} - k_t$) is equal to investment ($i_t = \gamma f(k_t)$) minus depreciation (δk_t). We have a steady state when $\Delta k_t = 0$, i.e. $\gamma f(k^{ss}) = \delta k^{ss}$, as illustrated below:

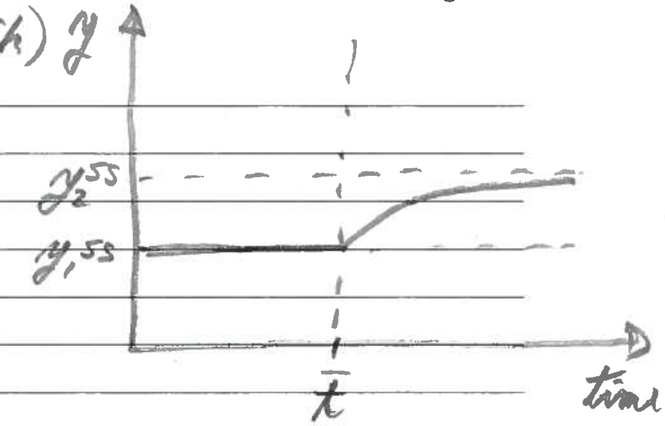


k^{ss} is a steady-state capital stock because $i = d$ at point A.

Since depreciation is a straight line, the existence of a steady-state is insured by the fact that production exhibits decreasing returns to capital, i.e. the slope of $f(k)$ decreases with k.

c) (10 points) Suppose that up to time $\bar{t}$, the economy was operating at its long-run steady state values corresponding to an investment rate γ_1. At $\bar{t}$, the saving rate suddenly jumps to $\gamma_2 > \gamma_1$. Draw a graphic describing the evolution of income per worker *over time* and discuss the effects on the growth rate.





Before time $\bar{t}$, income is at steady-state level $y_1^{ss} = f(k_1^{ss})$. When the investment rate jumps to γ_2, investment exceeds depreciation and the capital stock increases to eventually reach a new steady-state value $k_2^{ss} > k_1^{ss}$ and corresponding income level $y_2^{ss} > y_1^{ss}$.

We see that during the transition between the two steady-state, there is an income growth phase which gradually diminishes to 0.