PROBLEMS ON EXPECTATIONS
BASED ON CLASS NOTES DOCUMENT ENTITLED
Expectations and the demand for domestic goods

(1) Calculate the *yield to maturity* for the following bonds that promise to pay $ 100 at maturity:

a) Its maturity is 5 years (n=5) and its current price is $ 90.
LET p_{nt} DENOTE THE PRICE OF A BOND AT YEAR t WITH A MATURITY OF n YEARS AND i_{nt} BE THE YIELD TO MATURITY. WE HAVE

$$p_{nt} = \frac{100}{(1+i_{nt})^n} \Rightarrow i_{nt} = \left(\frac{100}{p_{nt}}\right)^{\frac{1}{n}} - 1.$$

HENCE, THE YIELD TO MATURITY IS

$$i_{5t} = \left(\frac{100}{90}\right)^{\frac{1}{5}} - 1 = 2.13\%$$

b) The maturity is 5 years and the current price is $ 80.

$$i_{5t} = \left(\frac{100}{80}\right)^{\frac{1}{5}} - 1 = 4.56\%$$

A decrease in the price of a bond is associated with an increase in its yield, all else equal.

c) The maturity is 10 years and the current price is $ 90.

$$i_{10t} = \left(\frac{100}{90}\right)^{\frac{1}{10}} - 1 = 1.06\%$$

d) What does the above say about the effect of a decrease in the price of a bond on its yield, *all else equal*.

(2) Suppose that the financial data is provided today in the newspapers:

Maturity (n years)	1	2	3	4	5
Yield to maturity (i_{nt}%)	1	1.5	2.0	2.5	3.0

(a) Calculate the implication regarding the one-year interest rate that markets expect to prevail during year $t+1$ (i_{12})?
THE EXPRESSION LINKING YIELDS TO MATURITY WITH SHORT-TERM INTEREST RATES IS

$$i_{n,t} = \frac{1}{n}(i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + i^e_{1,t+n-1}).$$

Here we have

$$i_{2t} = \frac{1}{2}(i_{1t} + i^e_{1,t+1}) \Rightarrow 1.5 = 0.5(1.0 + i^e_{1,t+1}) \Rightarrow i^e_{1,t+1} = 2.0\%$$

Markets therefore expect the short-term interest rate to increase from 1.0% at t to 2.0% at $t+1$.

(b) Calculate the implication regarding the one-year interest rate that markets expect to prevail between years 4 and 5 (i_{12})?
We have

$$i_{5t} = \frac{1}{5}(i_{1t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + i^e_{1,t+4})$$

and

$$i_{4t} = \frac{1}{4}(i_{1t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3}).$$

This can be expressed as

$$5i_{5t} = i_{1t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + i^e_{1,t+4}$$

and

$$4i_{4t} = i_{1t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3}.$$

Taking the difference gives

$$i^e_{1t+4} = 5i_{5t} - 4i_{4t} = 5 \times 3 - 4 \times 2.5 = 5\%.$$

Markets expect the one-year interest rate to increase from 1% now to 5% in five years.

(3) **On bond pricing** Suppose that we have the following data today (year t) regarding expected short-term interest rates:

	i_{1t}	i^e_{1t+1}	i^e_{1t+2}	i^e_{1t+3}
Expected yield (one year %)	4.0	3.0	2.0	1.0

(a) Calculate the price (p_{2t}) of a bond that promises to pay \$100 in two years. The price of a two-year bond is given by

$$p_{2t} = \frac{100}{(1 + i_{1t})(1 + i^e_{1t+1})}.$$

We thus have

$$p_{2t} = \frac{100}{(1.04)(1.03)} = \$93.35$$

(b) Calculate the price (p_{4t}) of a bond that promises to pay \$100 in four years. Using the same method as above with an additional two years of discounting gives

$$p_{2t} = \frac{100}{(1.04)(1.03)(1.02)(1.01)} = \$90.62$$

(c) Assume that an announcement by the bank of Canada leads financial markets to revise upward their expectations regarding short-term interest rate three and four years from now, say at $i^e_{1t+2} = 3.0$ and $i^e_{1t+3} = 2.0$. All else remains the same. Calculate the implication on the price of a bond (p_{4t}) that promises to pay \$100 in four years. What does this say about the effect of an increase in expected short-term interest rates on bond prices? With the new interest rates, we now have

$$p_{2t} = \frac{100}{(1.04)(1.03)(1.03)(1.02)} = \$88.85$$

We therefore conclude that increases in the expected future interest rates tends to lower the value of today's bonds, all else equal.

(4) **On present-discounted values and investment decisions** Jesse and Wally are new business partners who confider building a Brownie factory. The following table gives the net profits that they expect to receive at the each of each year with certainty. The yield to maturity of a four-year bond is presently $i_{4t} = 3\%$. Assume that this 3% also corresponds to the one-year expected interest rates over the four years. There is no inflation and the factory becomes obsolete after its fourth year of operation.

	π^e_t	π^e_{t+1}	π^e_{1t+2}	π^e_{1t+3}
Expected net profit	100	150	200	200

(a) What is the maximum price that Jesse and Wally are willing to pay in order to build the factory? The maximum building price must correspond to the present value of the expected stream of net profits. This is given by

$$V_0 = \frac{100}{1.03} + \frac{150}{1.03^2} + \frac{200}{1.03^3} + \frac{200}{1.03^4} = 599.21$$

(b) Suppose that the expected, short-term one year interest rate for the fourth year increases from an initial value of $i_{1t+3} = 3\%$ to a new value of $i_{1t+3} = 5\%$. All else remains equal. Recalculate the maximum building cost required in order to implement the project. What does this say about the effect of an increase in expected short-term interest rates on the investment levels today? With

$i_{1t+3} = 5\%$, we now have

$$V_0' = \frac{100}{1.03} + \frac{150}{1.03^2} + \frac{200}{1.03^3} + \frac{200}{(1.03^3)(1.05)} = 595.82$$

This suggests that, all else equal, an increase in the expected future interest rates tends to decrease the willingness to pay for an investment today because it decreases the present discounted value of future net profits. As a result, higher expected short term interest rates will depress investment levels today.

(c) Suppose that a suddenly gloomier outlook leads Jesse and Wally to revise down their expected profits in the third and fourth year to decrease to $\pi^e_{1t+2} = \pi^e_{1t+3} = 150$. All else remains equal. Recalculate the maximum building cost required in order to implement the project. What does this say about the effect of a gloomier outlook about the future on the investment levels today? The maximum willingness to pay for building the factory is now

$$V''_0 = \frac{100}{1.03} + \frac{150}{1.03^2} + \frac{150}{1.03^3} + \frac{150}{1.03^4} = 509.02$$

This suggests that if a gloomier outlook about the future translate into lower future net profit levels, all else equal, this will decrease the willingness to pay for building the factory today and therefore depress the investment level in general.

(5) **On the permanent income hypothesis** Suppose that you are 20 years old today and expect to live until 90. You consider the future in terms of the seven decades to come. The following table gives the total gross labor income that you expect to make for each decade.

Decade	20s	30s	40s	50s	60s	70s	80s
Total gross income ($\times\$1,000$)	200	400	500	600	400	0	0

The tax rate on labor income is 30%. There is no inflation and the real interest rate is always zero. You just inherited \$50K from an uncle you have never met and intend to leave \$100K to your eventual kids when you die.

(a) Calculate your yearly permanent income. The permanent income is made up of the sum of human and non-human wealth divided by the number of years one will live. The human wealth is the discounted sum of labor income net of taxes. This gives

$$y = \frac{50 + 0.7(200 + 400 + 500 + 600 + 400) - 100}{70} = \frac{1420K}{70} = \$20,285.$$

Note that the amount \$1,420K corresponds to the value of your total wealth net of your bequest of \$100K.

(b) Suppose that you want to keep your consumption at the same level for all of your life. In which decade will you switch from being a net borrower to a net saver? In which decade will you switch from being a net debtor to a net creditor? If you spend \$20,285 per year during your 20s, you will spend a total of 202,850. Given that you begin with a human wealth of \$50K, you will need an extra 202,850 - 50,000 = \$152,850. Given that your after tax income is equal to 0.7(200)=140K, this means that you will need to borrow 152,850-140,000=\$12,850. You are therefore a *net borrower* in your 20s and begin your 30s with as a *net debtor* with a debt of \$12,850.
In your 30s, you spend a total of 202,850 also. Your total disposable income is 0.7(400K)=280,000. You therefore save 280,000-202,850=\$77,150. You therefore become a *net saver* during your 30s. Once you pay off your debt also, you are left with a surplus of 77,150-12,850=\$64,300. Consequently, you also become a *net creditor* in your 30s.

(c) Suppose that work until 67. What will be the value of your accumulated assets when you start retirement at 68? Between the ages of 20 and 67 inclusively, you will spend a total of 48*\$20,285=\$973,680. Since your total wealth is \$1,420,000 (net of bequest), you accumulated assets at retirement have a value of 1,420,000-973,680=\$446,320. It can be verified that this will indeed allow you to keep on spending \$20,285 per year for the next 22 years until the age of 89.

(d) Suppose that you have a pleasant surprise when you begin work at 20 in the sense that you had underestimated your income for your 20s, which is now expected to be \$300K instead of \$200K. How does this 50% increase in your present income translate into an increase in your present consumption. Explain. Your total wealth net of bequest is now value at

$$50 + 0.7(300 + 400 + 500 + 600 + 400) - 100 = 50 + 1540 - 100 = 1490.$$

You will therefore spend 1490/70=\$21,286 per year. Since 21,286-20,285=\$1,001, a 50% increase in your present income has translated into a 4.93% increase in your present consumption spending. This corresponds to the permanent income hypothesis (or life-cycle theory of consumption), which holds that any increase

IN THE PRESENT INCOME LEVEL WILL LEAD TO A LOWER INCREASE IN CONSUMPTION SINCE IT WILL BE SPREAD OUT OVER ONE'S LIFETIME. IN PRACTICE, OF COURSE, THE SPREAD OVER ONE'S LIFETIME MAY NOT BE SO EQUAL AS ASSUMED HERE.

(6) **Interest rates and current consumption** Suppose that Penelope lives for two periods only, $t \in \{1, 2\}$. y_{dt} is her disposable income at period t and A_1 is her initial (non-human) wealth at period 1. She can save or borrow at interest rate r and cannot leave a bequest or unpaid debt after period 2. c_t is her consumption level at period t and s_1 represents her savings level in period 1. Penelope's preferences are such that she exhibits convex indifference curves between consumption levels in the two periods. (This problem also refers to the chapter 3 notes titled *A theory of intertemporal choice*.)

(a) Write down the two separate equations representing consumption levels at periods 1 and 2 respectively. (see mid-term 1 2014 part 1.a))

(b) Give the expression that represents Penelope's present-discounted value (PDV) of human wealth H_1. Give the expression that represents the PDV of her total wealth W_1. (see mid-term 1 2014 part 1.a)

(c) With the help of your answers in (6a), show that the intertemporal budget constraint is really just an equality between the PDV of consumption levels and the PDV of total wealth. (see mid-term 1 2014 part 1.a)

(d) With the help of a graphical analysis, concoct an example in which an increase in the rate of interest leads Penelope to *reduce* her present consumption level. AN INCREASE IN THE RATE OF INTEREST LEADS TO A STEEPER IBC. THIS IS ILLUSTRATED IN CASE A OF THE ACCOMPANYING GRAPHICS, WHERE POINT B DENOTES THE NO-SAVINGS, NO-BORROWING CONSUMPTION LEVELS. THE INITIAL CONSUMPTION LEVELS ARE GIVEN BY POINT A. WITH THE STEEPER IBC', THE NEW EQUILIBRIUM CONSUMPTION LEVELS ARE AT POINT A', IN WHICH THE CONSUMPTION LEVEL IN PERIOD 1, C_1^*, DECREASES TO C_1^*.

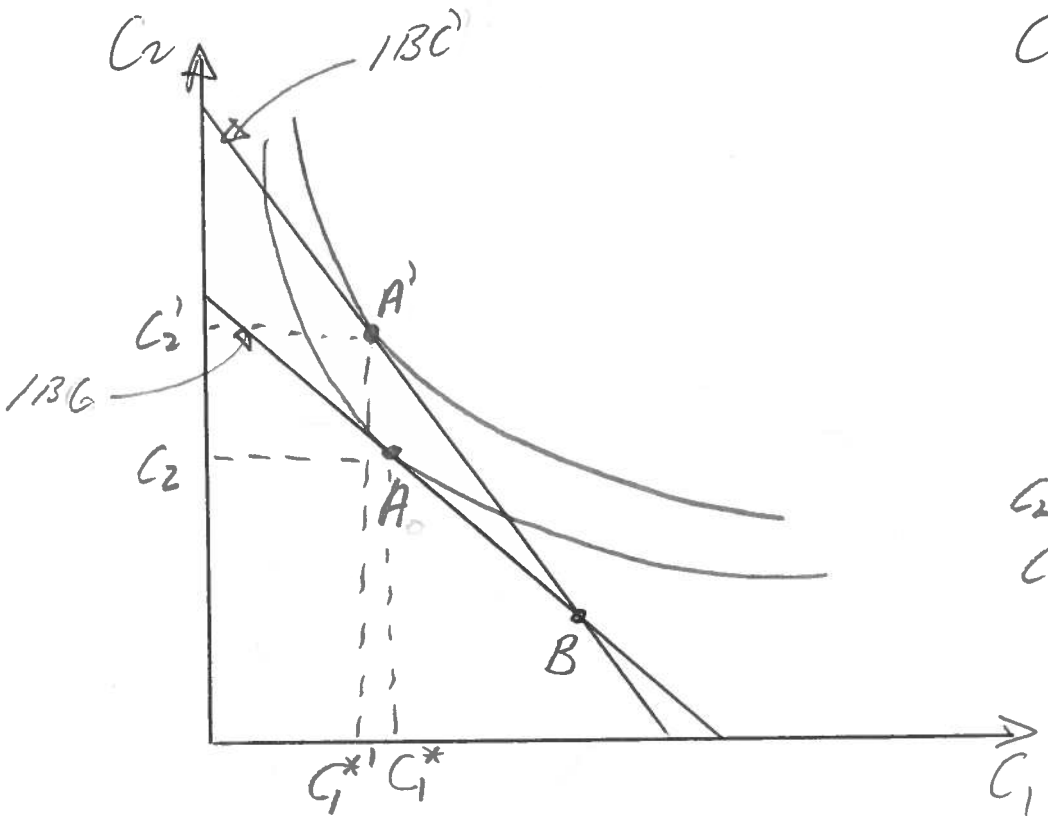
(e) With the help of a graphical analysis, concoct an example in which an increase in the rate of interest leads Penelope to *increase* her present consumption level. THIS IS ILLUSTRATED IN CASE B OF THE ACCOMPANYING GRAPHICS. WITH THE STEEPER IBC', THE NEW EQUILIBRIUM CONSUMPTION LEVELS ARE AT POINT A', IN WHICH THE CONSUMPTION LEVEL IN PERIOD 1, C_1^*, INCREASES TO C_1^*.

(f) Explain the difference between (6d) and (6e). The fact that an increase in the rate of interest has an ambiguous effect on the ppresent consumption level is due to its two opposite effects. One is called the *substitution effect*, which holds that with higher interest rates, the opportunity cost of today's consumption increases in units of tomorrow's consumption and this tends to lower today's consumption. The other is called the *wealth effect*, under which higher interest rates raises the total wealth value of net savers, who will tend to use it to increase their consumption levels at both periods through consumption smoothing. Which of the two effects prevails in practice is an empirical question.

(g) Which of the two possible effects of an increase in interest on present consumption levels is typically assumed to hold in the economy regarding the aggregate consumption level C_t? What does this say about the effect of an expansionary monetary policy (i.e., increased money supply through lower interest rates) by the central bank?
In the aggregate, the empirical evidence suggests that the substitution effect overtakes the wealth effect, i.e., an increase in the interest rate tends to lower present consumption. This implies that an expansionary monetary policy that reduces interest rates will be met with an increase in the present aggregate consumption level.

(h) Why are there some economists who have important reservations regarding the use of an expansionary monetary policy to stimulate the economy? An expansionary monetary policy implies that there is more money circulating and this may lead to additional inflation that may get out of hand later on.

CASE A


C_2
IBC'
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$C_1^{*\prime}$
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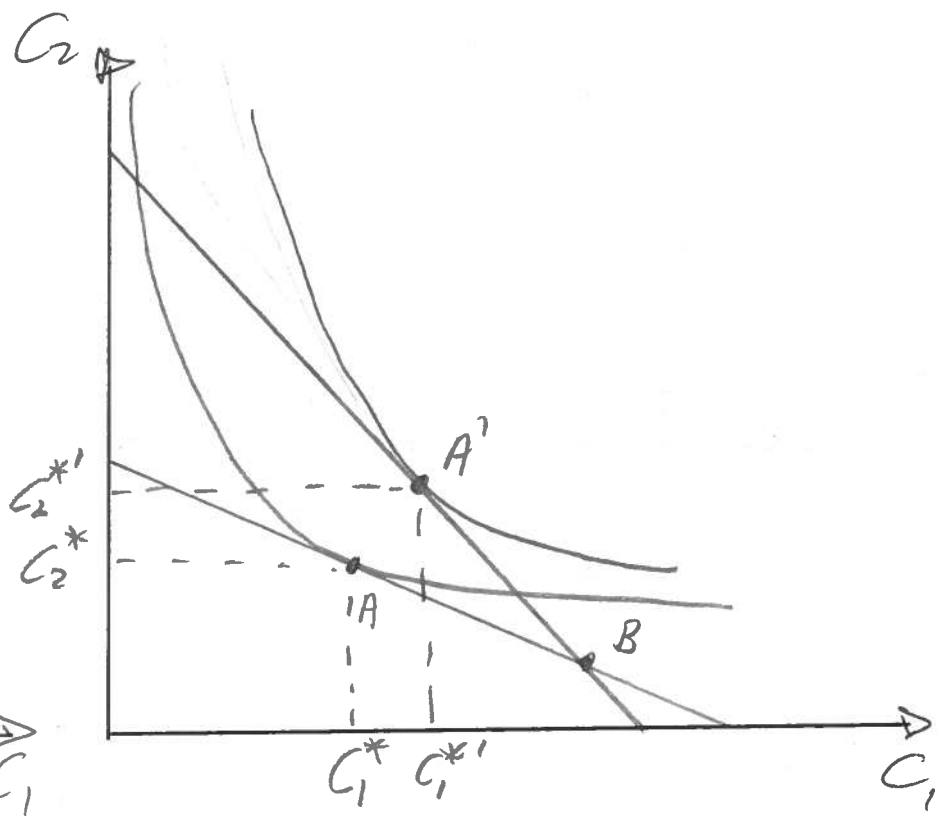
C_1

CASE B


C_2
$C_2^{*\prime}$
C_2^*
A'
A
B
C_1^*
$C_1^{*\prime}$
C_1