## PROBLEMS ON EXPECTATIONS BASED ON CLASS NOTES DOCUMENT ENTITLED Expectations and the demand for domestic goods

(1) On yield to maturity Calculate the yield to maturity for the following bonds that promise to pay $\$ 100$ at maturity:
a) The maturity is 5 years and the current price is $\$ 90$.
b) The maturity is 5 years and the current price is $\$ 80$. What does this say about the effect of a decrease in the price of a bond on its yield, all else equal?
c) The maturity is 10 years and the current price is $\$ 90$.
(2) On expected interest rates Suppose that the following financial data is provided today (year $t$ ) in the newspapers:

$$
\begin{array}{llcccc}
\text { Maturity ( } n \text { years) } & 1 & 2 & 3 & 4 & 5 \\
\text { Yield to maturity }\left(i_{n t} \%\right) & 1 & 1.5 & 2.0 & 2.5 & 3.0
\end{array}
$$

(a) Calculate the implication regarding the one-year interest rate that markets expect to prevail between years 1 and $2\left(i_{12}\right)$ ?
(b) Calculate the implication regarding the one-year interest rate that markets expect to prevail between years 4 and $5\left(i_{12}\right)$ ?
(c) What does the above say about the link between expected short term interest rates and the fact that yields to maturity increase with maturity?
(3) On bond pricing Suppose that we have the following data today (year $t$ ) regarding expected short-term interest rates:

$$
\begin{array}{lcccc} 
& i_{1 t} & i_{1 t+1}^{e} & i_{1 t+2}^{e} & i_{1 t+3}^{e} \\
\text { Expected yield (one year \%) } & 4.0 & 3.0 & 2.0 & 1.0
\end{array}
$$

(a) Calculate the price $\left(p_{2 t}\right)$ of a bond that promises to pay $\$ 100$ in two years.
(b) Calculate the price $\left(p_{4 t}\right)$ of a bond that promises to pay $\$ 100$ in four years.
(c) Assume that an announcement by the bank of Canada leads financial markets to revise upward their expectations regarding short-term interest rate three and four years from now, say at $i_{1 t+2}^{e}=3.0$ and $i_{1 t+3}^{e}=2.0$. All else remains the same. Calculate the implication on the price of a bond $\left(p_{4 t}\right)$ that promises to pay $\$ 100$ in four years. What does this say about the effect of an increase in expected short-term interest rates on bond prices?
(4) On present-discounted values and investment decisions Jesse and Wally are new business partners who confider building a Brownie factory. The following table
gives the net profits that they expect to receive at the each of each year with certainty. The yield to maturity of a four-year bond is presently $i_{4 t}=3 \%$. Assume that this $3 \%$ also corresponds to the one-year expected interest rates over the four years. There is no inflation and the factory becomes obsolete after its fourth year of operation.

(a) What is the maximum price that Jesse and Wally are willing to pay in order to build the factory?
(b) Suppose that the expected, short-term one year interest rate for the fourth year increases from an initial value of $i_{1 t+3}=3 \%$ to a new value of $i_{1 t+3}=5 \%$. All else remains equal. Recalculate the maximum building cost required in order to implement the project. What does this say about the effect of an increase in expected short-term interest rates on the investment levels today?
(c) Suppose that a suddenly gloomier outlook leads Jesse and Wally to revise down their expected profits in the third and fourth year to decrease to $\pi_{1 t+2}^{e}=\pi_{1 t+3}^{e}=$ 150. All else remains equal. Recalculate the maximum building cost required in order to implement the project. What does this say about the effect of a gloomier outlook about the future on the investment levels today?
(5) On the permanent income hypothesis Suppose that you are 20 years old today and expect to live until 90 . You consider the future in terms of the seven decades to come. The following table gives the total gross income that you expect to make for each decade.

| Decade | 20 s | 30 s | 40 s | 50 s | 60 s | 70 s | 80 s |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total gross income $(\times \$ 1,000)$ | 200 | 400 | 500 | 600 | 400 | 0 | 0 |

The tax rate on income is $30 \%$. There is no inflation and the real interest rate is always zero. You just inherited $\$ 50 \mathrm{~K}$ from an uncle you have never met and intend to leave $\$ 100 \mathrm{~K}$ to your eventual kids when you die.
(a) Calculate your yearly permanent income.
(b) Suppose that you want to keep your consumption at the same level for all of your life. In which decade will you switch from being a net borrower to a net saver? In which decade will you switch from being a net debtor to a net creditor?
(c) Suppose that work until 67 . What will be the value of your accumulated assets when you start retirement at 68 ?
(d) Suppose that you have a pleasant surprise when you begin work at 20 in the sense that you had underestimated your income for your 20s, which is now expected to
be $\$ 300 \mathrm{~K}$ instead of $\$ 200 \mathrm{~K}$. How does this $50 \%$ increase in your present income translate into an increase in your present consumption. Explain.
(6) Interest rates and current consumption Suppose that Penelope lives for two periods only, $t \in\{1,2\} . y_{d t}$ is her disposable income at period $t$ and $A_{1}$ is her initial (non-human) wealth at period 1. She can save or borrow at interest rate $r$ and cannot leave a bequest or unpaid debt after period 2. $c_{t}$ is her consumption level at period $t$ and $s_{1}$ represents her savings level in period 1. Penelope's preferences are such that she exhibits convex indifference curves between consumption levels in the two periods. (This problem also refers to the chapter 3 notes titled $A$ theory of intertemporal choice.)
(a) Write down the two separate equations representing consumption levels at periods 1 and 2 respectively.
(b) Give the expression that represents Penelope's present-discounted value (PDV) of human wealth $H_{1}$. Give the expression that represents the PDV of her total wealth $W_{1}$.
(c) With the help of your answers in (6a), show that the intertemporal budget constraint is really just an equality between the PDV of consumption levels and the PDV of total wealth.
(d) With the help of a graphical analysis, concoct an example in which an increase in the rate of interest leads Penelope to reduce her present consumption level.
(e) With the help of a graphical analysis, concoct an example in which an increase in the rate of interest leads Penelope to increase her present consumption level.
(f) Explain the difference between (6d) and (6e).
(g) Which of the two possible effects of an increase in interest on present consumption levels is typically assumed to hold in the economy regarding the aggregate consumption level $C_{t}$ ? What does this say about the effect of an expansionary monetary policy (i.e., increased money supply through lower interest rates) by the central bank?
(h) Why are there some economists who have important reservations regarding the use of an expansionary monetary policy to stimulate the economy?

