GROWTH ACCOUNTING

Growth accounting is used to measure the respective contributions of TFP growth and factor accumulation in total growth.

1 Solow's method (1957)

Solow did not explicitly account for human capital, as was common in those years because they did not typically realize its potential importance. We thus have $Y = AK^\alpha L^{1-\alpha}$ and we define output aggregate growth rate as

$$\hat{Y}_t \equiv \frac{Y_{t+1} - Y_t}{Y_t} = \frac{\Delta Y_t}{Y_t}. \tag{1}$$

Suppose L increases by 2% and salaries are equal to labor's marginal product, i.e. $w = MPL$. Total output will thus increase by $0.02 \cdot L \cdot w$. More generally, we have

$$\hat{Y} \equiv \frac{\Delta Y}{Y} = \frac{w \Delta L}{Y} \cdot \frac{L}{L} = \frac{wL}{Y} \cdot \frac{\Delta L}{L} = (1-\alpha)\hat{L}, \tag{2}$$

since wL/Y is the share of labor income in total income. Similarly for capital, we have

$$\hat{Y} = \frac{rK}{Y} \cdot \frac{\Delta K}{K} = \alpha \hat{K}. \tag{3}$$

Hence, for constant TFP, we have

$$\hat{Y} = \alpha \hat{K} + (1-\alpha)\hat{L}. \tag{4}$$

What if TFP changes also? Then $\hat{Y} > \alpha \hat{K} + (1-\alpha)\hat{L}$. But how so?

Math Review 1

$\|$*Let* $Z_0 = X_0 Y_0$.
Then $Z_1 = (X_0 + \Delta X_0)(Y_0 + \Delta Y_0)$.
$\Rightarrow \Delta Z_0 = X_0 Y_0 + X_0 \Delta Y_0 + \Delta X_0 Y_0 + \Delta X_0 \Delta Y_0 - X_0 Y_0$.
Since $\Delta X_0 \Delta Y_0 \approx 0$, *we have*

$$\hat{Z}_0 = \frac{X_0 \Delta Y_0}{X_0 Y_0} + \frac{\Delta X_0 Y_0}{X_0 Y_0} = \hat{X}_0 + \hat{Y}_0.$$

Rule to remember: If $Z = XY$, *then* $\hat{Z} = \hat{X} + \hat{Y}$. *And similarly, if* $X = Z/Y$, *then* $\hat{X} = \hat{Z} - \hat{Y}$.$\|$

With $Y = AK^{\alpha}L^{1-\alpha}$, we thus have

$$\hat{Y} = \hat{A} + \alpha\hat{K} + (1-\alpha)\hat{L}. \tag{5}$$

Since $\hat{Y}$, $\hat{K}$ and $\hat{L}$ are observable, we write

$$\hat{A} = \hat{Y} - \alpha\hat{K} - (1-\alpha)\hat{L}. \tag{6}$$

TFP growth is measured as a "left-over" difference between total growth and factor accumulation.

$$\text{SOLOW RESIDUAL } \equiv \hat{Y} - \left[\alpha\hat{K} + (1-\alpha)\hat{L}\right].$$

The part of total growth that cannot be observed must be due to productivity growth. But is it important?

2 What about labor productivity growth?

LP is the part of total growth that exceeds growth in the size of the labor force. From (5), we have

$$\hat{LP} = \hat{Y} - \hat{L} = \hat{A} + \alpha\hat{K} - \alpha\hat{L}. \tag{7}$$

Why does $\hat{L}$ have a negative effect on $\hat{LP}$? The capital dilution effect.

3 What about income per capita?

$$y = \frac{Y}{L} \Rightarrow \hat{y} = \hat{Y} - \hat{L}$$
$$k = \frac{K}{L} \Rightarrow \hat{k} = \hat{K} - \hat{L}$$

Hence, from (7),

$$\hat{y} = \hat{A} + \alpha\hat{k}$$
$$\Rightarrow \hat{A} = \hat{y} - \alpha\hat{k}.$$

4 What about human capital?

Let us simply replace L with hL. We have $Y = AK^{\alpha}(hL)^{1-\alpha}$. Hence

$$\hat{Y} = \hat{A} + \alpha\hat{K} + (1-\alpha)(\hat{h} + \hat{L})$$
$$\Rightarrow \hat{Y} - \hat{L} \equiv \hat{y} = \hat{A} + \alpha\hat{K} + (1-\alpha)\hat{h} - \alpha\hat{L}$$
$$\Rightarrow \hat{y} = \hat{A} + \alpha\hat{k} + (1-\alpha)\hat{h}$$

As a result, we have the following Solow residual augmented to account for human capital

$$\hat{A} = \hat{y} - \alpha\hat{k} - (1-\alpha)\hat{h}. \tag{8}$$

Again, TFP growth can be measured as a left-over component after factor accumulation has been accounted for.

For instance, if $\hat{y} = \alpha\hat{k} + (1-\alpha)\hat{h}$, then there is no TFP growth. The country *as a whole* is not learning anything new, i.e. no knowledge creation, learning-by-doing or new product development is going on. This is worrisome, even if $\hat{y}$ is now high. Why? Because factor accumulation is subject to diminishing returns. Hence, growth based on factor accumulation will surely slow down and stop in the long run.

Differences in growth experiences in the USSR and the USA during the 20th century are largely explained this way. But those are just the *proximate* causes of growth differences. The fundamental, most interesting and challenging question is: *Why was the creation of ideas so much lower in the USSR than the USA?* We will try to answer this later. But try to think about it now. If you can answer this, you've probably come a long way to understand why we are so much wealthier than our forebears; and why today, some countries are poor and others are rich. In the mean time, note how this analysis about proximate causes is a *necessary* first step towards uncovering the fundamental causes of income differences across the world. In the 1950s, many development experts thought that investments in physical capital was the key ingredient to put poor countries on track for sustained economic development. They were "mostly" wrong.