

Expectations and the demand for domestic goods

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1 Introduction

Recall that national output is given by the following expression:

$$Y_t = C_t + I_t + G_t + (EX_t - IM_t). \quad (1)$$

The right-hand side of the above equality denotes the *demand for domestic goods*. In this section and the next, we wish to look at what determines consumption (C_t) and investment (I_t) values *today* when agents are *forward-looking*, i.e. their present decisions are influenced by what they expect the future to be like. Besides the fact that the question is interesting for its own sake in order to understand the workings of our economy, the question is also of fundamental importance for policy purposes. Indeed, one the most controversial issue in economics is whether governments can kick start a sluggish economy through government spending. In a nutshell, the question can be formulated as follows:

If GDP growth is negative at period t because consumption (C_t) and investment (I_t) have decreased, can this be compensated for by an increase in government expenditures (G_t)?

A quick glance at expression (1) gives the impression that the answer should be yes. The problem is that the expression gives a purely static view of the relation between GDP and the demand for domestic goods. As we are about to see, the reality is more complicated because present consumption and investment decisions are affected by expectations about the future and present government expenditures affect those expectations.

We begin by reviewing some basic concepts in finance with a look at bond pricing. This is followed by a study of the determinants of the demand for domestic goods, i.e., investments and consumption. We then conclude by a discussion about expectations as a prelude to the slide presentation on the stimulus package adopted in Canada following the 2008-2009 recession.

2 The value of a bond

A bond is a financial instrument in which the *bond issuer* promises to make some future payments to the *bond holder* in return for a present payment by the holder. This way, the bond issuer borrows money from the bond holder, who respectively become the *debtor* (borrower) and the *creditor* (lender). Future payments make take many forms and usually include periodic interest payments and a final payment equal to the initial amount lent (called the principal, the face value, or the nominal price). The *maturity date* is the date at which the final payment is being made.

Bonds are usually liquid assets, i.e., they can be bought and sold before reaching their maturity date. The current price of a bond will depend in part on the amounts left to be paid by the issuer and will generally differ from its nominal value, as will be seen below.

2.1 A one year maturity bond

The maturity of a bond corresponds to the time at which it is making its last payment. In order to simplify, we consider bonds that promise to pay \$100 at maturity without any other intermediate payment. Let p_{st} denote the price at period t of a bond that matures at year $t + s$. In what follows, we shall refer to period t as the current period. The price at time t of a bond that matures in one year is thus denoted p_{1t} and its current *return* is expressed as:

$$i_{1t} = \frac{\$100 - p_{1t}}{p_{1t}}. \quad (2)$$

i_{1t} is also referred to as the (current) *one-year nominal interest rate* or as the *yield*. Another way to look at equation (2) is to assume that a bond has a yield of i_{1t} such that its price is given by

$$p_{1t} = \frac{\$100}{1 + i_{1t}} < 100. \quad (3)$$

Note that because bonds provide a positive return, i.e. $i_{1t} > 0$, one dollar tomorrow is worth less than one dollar today. Said otherwise, given that government bonds provide a return of $i_{1t} > 0$, one is indifferent between receiving $p_{1t} < 100$ today or receiving \$100 tomorrow. For this reason, p_{1t} is called the *present value* equivalent of \$100 next year given that i_{1t} is the yield on a bond. Indeed, we have

$$p_{1t}(1 + i_{1t}) = \$100 \quad (4)$$

The above equality really represents a *no-arbitrage condition*. To see why, suppose that you have a choice between two bonds: bond *A* provides a return i_{1t} while bond *B* promises to pay \$100 in one year and costs p_{1t} today. If $p_{1t}(1 + i_{1t}) > \$100$, then no-one would want to buy bond *B* at price p_{1t} because bond *A* provides a higher payment than \$100 one year from now. Those trying to sell would thus have to lower their price until the equality is re-established. The opposite would occur if the inequality goes the other way, thus driving up the price of bond *B* today.

2.2 A two-year maturity bond

Suppose now that bond *B* promises to pay \$100 two years from now. Its current price is denoted p_{2t} and you want to hold the bond for one year only. You expect to sell the bond at price $p_{1,t+1}^e$ one year on, as this represents the price of a one-year maturity bond at time $t + 1$ that promises to pay \$100 one year later. Superscript e denotes the fact that now, the selling price is an expected price. Bond *A* still provides a one-year return i_{1t} . The no-arbitrage condition requires the following equality to hold (for the same reason seen in (4) above):

$$p_{2t}(1 + i_{1t}) = p_{1,t+1}^e. \quad (5)$$

But how is $p_{1,t+1}^e$ determined? The answer depends on the expected yield of a one-year bond during period $t + 1$, denoted $i_{1,t+1}^e$. The no-arbitrage condition

for such a bond requires the following (see figure 1 for an illustration of the time schedule):

$$p_{1,t+1}^e(1 + i_{1,t+1}^e) = \$100. \quad (6)$$

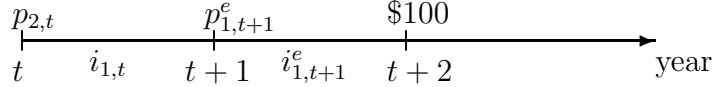


Figure 1: Two-year maturity bond

Combining expressions (5) and (6) gives the following equality:

$$p_{2t} = \frac{\$100}{(1 + i_{1t})(1 + i_{1,t+1}^e)}. \quad (7)$$

The above expression for p_{2t} denotes the present-value of a two-year bond that promises to pay \$100 at $t + 2$.

2.3 Bond prices and yields

We call the *yield to maturity* the constant, or average, yearly interest rate that corresponds to the bond price and its future payments, denoted i_{st} . In the case of the above two-year bond, we thus have:

$$p_{2t} = \frac{100}{(1 + i_{2t})^2}. \quad (8)$$

Suppose that $p_{2t} = \$95$. Then $i_{2t} = 2.6\%$. Putting together expressions (7) and (8), we have $(1 + i_{2t})^2 = (1 + i_{1t})(1 + i_{1,t+1}^e)$, which can be usefully approximated by $i_{2t} \approx (1/2)(i_{1,t} + i_{1,t+1}^e)$. More generally, for a bond with n year maturity:

$$i_{n,t} = \frac{1}{n}(i_{1,t} + i_{1,t+1}^e + i_{1,t+2}^e + \dots + i_{1,t+n-1}^e). \quad (9)$$

This means that if short-term future interest rates are expected to increase, then bonds with a longer maturity will yield a higher yield to maturity than shorter maturity bonds. Note that the higher yield of longer maturity bonds

is not necessarily due to the fact that they are considered more risky in terms of the probability of repayment in the far future. It may just reflect the present circumstances of the economy which lead investors to believe that interest rates will go up, say because of expectations about a contractionary monetary policy in the future. This is typically illustrated by the **yield curve**, which plots the yield to maturity against the maturity of the bonds, as illustrated in figure 2. (See file `Gvmt of Canada bond yield curve(G&M2014).pdf` for a real-life equivalent in the Globe and Mail.)

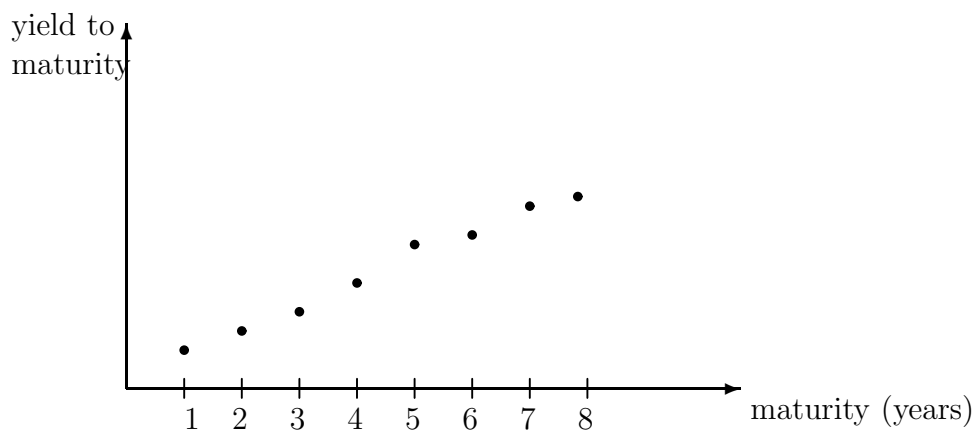


Figure 2: A fictitious increasing yield curve

The data points in figure 2 are observable daily in the financial press, as they report on the price of bonds with various maturity dates, and thus their yield to maturity can be inferred. If the yield curve turns out to be downward slopping, one infers that investors are expecting short-term nominal interest rates to be going down. Suppose for instance that given their present price and promises of payments, the yield to maturity of one-year and two-year bonds are equal to 3% and 2% respectively. From expression (9), we have $2 = (1/2)(3 + i_{1,t+1}^e)$ and thus $i_{1,t+1}^e = 1\%$: investors expect the one-year nominal interest rate to decrease.

3 The decision to invest

3.1 A simple three-period example

Suppose that you have a specific project which consists in building a factory that costs \$1 million. The project is expected to bring the following flows of profits in the next three periods: π_1^e , π_2^e and π_3^e . Should you build that factory? The answer to this question hinges on the opportunity cost of the \$1 million being set aside for that project, i.e., it must be compared with the best alternative use of that money. To simplify things, suppose that the best you can do with that money is to use it to buy government bonds. The expected yearly returns from bonds over the next three years is given by i_1 , i_2^e and i_3^e . The timing of project profits and bond returns is illustrated in figure 3.

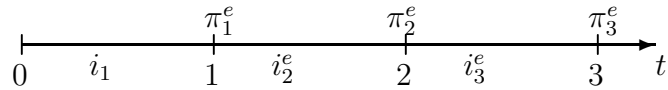


Figure 3: A three-period investment problem

If your objective is to cash in on the investment after three periods, then the bond investment will leave you with the following amount:

$$V_3^B = \$10^6(1 + i_{10})(1 + i_{11}^e)(1 + i_{12}^e) \quad (10)$$

In order to compare the above value with that of the factory, we must assume that the profit flows that accrue at the end of every year are being reinvested into bonds. For instance, year one profit flow π_1^e will provide amount $\pi_1^e(1 + i_{11}^e)(1 + i_{12}^e)$ if reinvested and cashed in at period 3. Applying this to the following two profit flows yields the following amount that can be cashed in at period 3:

$$V_3^P = \pi_1^e(1 + i_{11}^e)(1 + i_{12}^e) + \pi_2^e(1 + i_{12}^e) + \pi_3^e. \quad (11)$$

Note that the above assumes that the factory has no resale value at period 3. You will therefore prefer to build the factory if $V_3^P > V_3^B$. Combining this inequality with expressions (10) and (11), we have:

$$\$10^6 < \frac{\pi_1^e}{(1 + i_{10})} + \frac{\pi_2^e}{(1 + i_{10})(1 + i_{11}^e)} + \frac{\pi_3^e}{(1 + i_{10})(1 + i_{11}^e)(1 + i_{12}^e)}. \quad (12)$$

The right-hand side of the above represents the present-value of the future profit flows from the project, where the discount rate being used corresponds to the expected return from the best alternative use of the money to be invested. The inequality says that the project should be implemented if the present value of future profit flows exceeds the initial outlay.

The above example assumed from the outset that the person who wants to build the \$1m factory is the same person that has \$1m of savings to invest. This was made as a simplification. But it is important to realize that the same result of inequality (12) holds even if savers and investors are not the same individuals. To see why, suppose that you still have the idea of building the factory but have no savings of your own to be used to implement the project. One thing that you can do is to borrow the money from a bank. If the best that the bank can do with the depositors' savings is also to place it into bonds that yield the same return as per equation (10), then all the bank will require is that you pay back the amount V_3^B at period 3. Consequently, as an investor, you will still decide to implement the project if $V_3^P > V_3^B$, i.e. the decision is independent of whether investors and savers are the same individuals or not.

One can see here a fundamental role played by banks: they act as intermediaries between people who want to save money but have no good project to implement and those who have ideas about good projects but do not have money to invest. Banks allow for both to “meet”.

3.2 An infinite horizon investment problem

Most projects last more than three years. In this section, we consider a project which is expected to yield a constant yearly stream of profits π per unit of capital into the indefinite future, i.e. $\pi_1^e = \pi_2^e = \pi_3^e = \dots \equiv \pi$. One unit of capital costs \$1. We similarly assume a constant yearly expected interest rate equal to i , i.e., $i_{11} = i_{12}^e = i_{13}^e = \dots \equiv i$. Following the same procedure as for the three period investment problem above, the present value of the project can be expressed as follows:

$$V_0 = \frac{\pi}{1+i} + \frac{\pi}{(1+i)^2} + \frac{\pi}{(1+i)^3} + \dots \quad (13)$$

Multiplying both sides of the above by $1/(1+i)$ gives:

$$\frac{1}{1+i}V_0 = \frac{\pi}{(1+i)^2} + \frac{\pi}{(1+i)^3} + \dots \quad (14)$$

Subtracting expression (14) from (13) gives:

$$\left(1 - \frac{1}{1+i}\right) V_0 = \frac{\pi}{1+i}. \quad (15)$$

Simplifying yields the following simple expression for the present value of the project:

$$V_0 = \frac{\pi}{i}. \quad (16)$$

As expected, expression (16) implies that the present value of a project increases with its expected future profit stream and decreases with the rate of interest on bonds. An additional unit of capital will be built only if V_0 is larger than its cost today of \$1. Generally, we conclude that *all else equal*, the investment level in the economy will increase when investors expect higher future profits. And conversely, the investment level will go down if, all else equal, investors expect higher interest rates. We can represent this result as follows:

$$I_t = I(\overset{+}{\pi^e}, \overset{-}{i^e}). \quad (17)$$

3.3 An infinite horizon investment problem with depreciation

Suppose now that at every period, a proportion δ of the factory's capital stock becomes obsolete due to deterioration. At any period, the profit *per unit of capital* remains constant over time and equal to π . Then the expected profit at time $t+1$ from a unit of capital built today equals fraction $1-\delta$ of the profit produced at time t , i.e $\pi_2 = (1-\delta)\pi_1$, $\pi_3 = (1-\delta)\pi_2 = (1-\delta)^2\pi_1$, and so on. We have:

$$V_0 = \frac{\pi}{1+i} + \frac{(1-\delta)\pi}{(1+i)^2} + \frac{(1-\delta)^2\pi}{(1+i)^3} + \dots \quad (18)$$

Multiplying through by $(1-\delta)/(1+i)$ gives:

$$\frac{1-\delta}{1+i} V_0 = \frac{(1-\delta)\pi}{(1+i)^2} + \frac{(1-\delta)^2\pi}{(1+i)^3} + \dots \quad (19)$$

Subtracting (19) from (18) and rearranging gives:

$$V_0 = \frac{\pi}{i+\delta}. \quad (20)$$

Depreciation has the same depressing effect on investment as the discount rate i .

3.4 An infinite horizon investment problem with uncertain property rights

As a last application, suppose now that property rights over the factory are uncertain in the following sense: At any period t , there is a probability $\theta \in (0, 1)$ that the factory owners be expropriated by the government authorities. When expropriation occurs at t , initial investors lose all claims to profit streams from period $t + 1$ and after. This means that if today's investor expect a profit of π_2 at period 2, then the probability that he/she will pocket that profit is equal to $1 - \theta$, thus yielding an expected profit of $(1 - \theta)\pi_2$. Similarly, the probability of being able to receive profit π_3 at period 3 is given by $(1 - \theta)^2$, which accounts for the fact that an expropriation could have occurred during period 2 or during period 3. The present value of one unit of capital built today is thus given by:

$$V_0 = \frac{\pi}{1+i} + \frac{(1-\theta)\pi}{(1+i)^2} + \frac{(1-\theta)^2\pi}{(1+i)^3} + \dots \quad (21)$$

We note that the above expression is the same as expression (18), where θ replaces δ . Hence, a probability of expropriation affects investment decisions in a similar manner as a depreciation rate on capital, which was seen above to have an effect akin to a higher discount rate. Not surprisingly, introducing uncertainty of property rights tends to lower the present value of an investment project. As discussed in the study on long run growth, this is consistent with the fact that respect for property rights is an important determinant of long-run growth.

4 Consumption decisions

We now turn to another determinant of the demand for domestic goods, the consumption level C_t .

Just like investment decisions, today's consumption level is also a *forward-looking decision* as it is based, at least partly, on our expectations about future income levels. Using only current income levels to determine how much people consume today is not realistic. To see why, let us go through the following example.

Consumption and lifetime income You are now 19 years old and starting university. You will not work while studying. Your plan is to graduate in four years from now, take a year off to travel the world, and then start

working at age 24 with a starting wage of \$ 50,000. Abstracting from inflation consideration to simplify things, you expect your wage to increase by 2% per year until retirement at age 65. You expect to live until age 85. Your tax rate will be 30% throughout your lifetime. With this information, you can calculate your **human wealth** H_0 , i.e., the present discounted value of your lifetime *labor* income. To simplify, we assume a zero discount rate, i.e., a dollar spent today is worth the same as a dollar spent 30 years from now. We have:¹

$$H_0 = 0.7(\$50,000)(1 + 1.03 + 1.02^2 + 1.02^3 + \dots + 1.02^{41}) = \$2,270,178. \quad (25)$$

Since you have 67 years left to live, you could potentially decide to spend \$2,270,178/67, or \$33,883, per year starting now at age 19. For this reason, we say that \$33,883 corresponds to your **permanent income** level. Note that in this calculation, we assume that you have no other sources of income or wealth than your labor income, i.e. you will not receive an income from some capital investment, nor will you receive an inheritance, etc.

In order to consume \$33,883 per year, you would have to borrow five times that amount before you begin to work at age 24, i.e. \$169,416. Most students would not borrow so much money. This could be because of:

- i) uncertainties over their future income levels;
- ii) a preference to spend more later when they have children;
- iii) the bank's unwillingness to lend the money;
- iv) people not being so forward looking.

¹The value of a stream of income y that increases at constant rate $s\%$ per year over n years is given by:

$$V_0 = y(1 + (1 + s) + (1 + s)^2 + (1 + s)^3 + \dots + (1 + s)^{n-1}). \quad (22)$$

Multiplying both sides by $1 + s$ gives:

$$(1 + s)V_0 = y((1 + s) + (1 + s)^2 + (1 + s)^3 + \dots + (1 + s)^{n-1} + (1 + s)^n). \quad (23)$$

Subtracting (23) from (22) and rearranging gives:

$$V_0 = y \frac{1 - (1 + s)^n}{1 - (1 + s)}. \quad (24)$$

This is how we obtain the figure for W_0 in expression (25).

In any case, even though your current consumption level may not correspond one-for-one with your human wealth as defined above, it is reasonable to assume that current consumption is closely correlated with people's human wealth, which in turn is determined by people's expectations about their future disposable income levels.

Of course, one should add the fact that people's income also depends on other types of assets, such as savings accounts and bonds, ownership shares in companies, real estate, cars, land, etc. Lifetime income levels will also depend on those non-human assets. We thus denote the **total wealth** of an individual as W_t , which comprises the present value of expected incomes from all sources, both labor and non-human. Note that expectations about future profit levels affect the present value of non-human wealth and thus will affect present consumption decisions as well as investment decisions.

Since it is also reasonable to assume that current wages determine current consumption levels also, the consumption level can be represented by the following expression:

$$C_t = C(W_t^{(+)}, y_t^{(+)}, T_t^{(-)}), \quad (26)$$

where y_t and T_t respectively denote the current income level and the current tax level. Keep in mind that total wealth W_t depends on expected future income levels and tax levels. In the literature, the derivation of expression (26) as a determinant of current consumption levels is referred to as the **permanent income hypothesis** or the **life-cycle theory of consumption**. The upshot is that expectations about the future play a major role in consumers' decisions today, a point to which we next turn.

5 Expectations and government policy

We have seen that consumption C_t and investment I_t decisions are both based on expectations about

- i) Future profit levels
- ii) Future interest rates
- iii) Future tax rates

This implies that consumption and investment will typically move in tandem. Investment is however more volatile than consumption as it is easier for a firm to delay investment entirely than for consumers to delay consumption entirely. On the other hand, investment makes up a much smaller percentage of GDP than consumption: In Canada in 2011, we have $I_t/Y_t \approx 23\%$ while $C_t/Y_t \approx 57\%$. In the end, therefore, investment and consumption tend to contribute about equally to short-term fluctuations.

We can now return to our initial question:

Can a drop in both consumption C_t and investment I_t be compensated for by an increase in government spending G_t ?

This is a difficult question that is far from resolved among economists. Indeed, there is little convincing empirical evidence either way. What I propose to do instead is to at least provide a good way to think about the issues. To this end, I first introduce below a few theoretical concepts. Those concepts will then be applied to some case studies in the accompanying slide presentation titled “Government and fiscal stimulus”.

The first place to start is to consider how the increase in G_t is being paid for, i.e. whether it is deficit-financed or not.

The no-deficit case If the government decides to increase G_t without running an additional deficit, it will then have to increase current taxes T_t by an equivalent amount ($\Delta^+G_t \Rightarrow \Delta^+T_t$). This means that the increase in G_t is likely to be directly compensated for by a drop in current consumption and investment. For this reason, a no-deficit increase in government expenditures is generally not considered a good way to kick-start a sluggish economy. This is rather uncontroversial. In fact, it is rather considered contractionary, i.e., likely to contribute to even lower GDP growth.

The deficit-financed case In this case, the current increase in G_t is funded by additional government borrowing to be repaid for in the future. There are really two ways to repay that debt in the future: increase future tax rates and/or increase the money supply (print more money) ($\Delta^+G_t \Rightarrow \Delta^+T_{t+1}$ and/or Δ^+M_{t+1}). Resorting to future tax increases or money supply is considered to have various effects on the economy, summarized as follows:

- If people expect future taxes to increase, then their present total wealth will decrease, thus causing current consumption and investment to de-

crease ($\Delta^+T_{t+1} \Rightarrow \Delta^-W_t \Rightarrow \Delta^-C_t$ and Δ^-I_t). This argument is based on the permanent income hypothesis. Some economists believe that this effect is strong enough to almost completely crowd out the stimulating effect of an increase in current government spending.

- If people expect the money supply to increase in the future, they will expect inflation to pick up. All else equal, this may increase current investment as it makes it easier to reimburse a given debt. As will be discussed in the accompanying slides, this was an important argument during the great depression of the 1930s in the USA when there was deflation (falling prices). The drawback is that some economists believe that the problem of inflation may get out of hand in the future.

In the literature, the debate about whether increased government spending can stimulate the economy is often summarized by the value of the **multiplier effect**, i.e.,

How does \$1 of current extra government spending ($\Delta^+G_t = \$1$) translate into an increase in current GDP (Δ^+Y_t)?

If $\Delta^+Y_t = \$1.5$, then the multiplier is equal to 1.5. If $\Delta^+Y_t = \$0.5$, then the multiplier is equal to 0.5. Stimulating the economy through Δ^+G_t is not considered worthwhile if the multiplier value is near 0. The problem is that there are large discrepancies between economists about what the actual value of the multiplier is. In the midst of the 2008 great recession in the USA, some argued that it was above 1 while others believed that it was almost 0!

It should be noted, finally, that many economists are wary of increased government spending because it invariably comes with increased government intervention in the economy and thus, increased opportunities for corruption and rent seeking.

We now turn to the slide presentation on the role of expectations which presents case studies.