The choice between consumption and saving
A theory of intertemporal choice

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1 Introduction

We have seen that private consumption made up about 60% of all expenditures in the economy in Canada in 2010. Understanding consumption decisions is therefore fundamental to understanding GDP variations in the short run. And since consumption decisions are closely linked to saving decisions, understanding consumption decisions is also fundamental to explain long run growth.

2 The Theory of Intertemporal Choice

2.1 Assumptions

1. The consumer lives two periods only, 1 and 2.

2. Y_d denotes a consumer's disposable income, i.e., her total income net of all taxes. We have Y_{d1} and Y_{d2} for each period respectively.

3. W_1 is initial wealth at period 1.

4. The consumer can save or borrow at rate interest r. Saving is denoted S_1 and corresponds to borrowing if negative.

5. The consumer chooses consumption levels at periods 1 and 2, resp. C_1 and C_2.

6. Assumptions about consumer preferences will be described in section 2.3.

7. A consumer cannot leave a bequest or unpaid debt after period 2.

2.2 The intertemporal budget constraint (IBC)

Consumption levels at periods 1 and 2 are given by the following:

$$C_1 = W_1 + Y_{1d} - S_1 \tag{1}$$
$$C_2 = Y_{2d} + (1+r)S_1 \tag{2}$$

Combining the above two equations gives:

$$C_2 = (1+r)(W_1 + Y_{1d} - C_1) + Y_{2d} \tag{3}$$

The above equation represents an *intertemporal budget constraint* since it uniquely links C_2 to C_1. Any choice of C_1 yields one specific value for C_2, as represented by figure 1 where we have assumed the following values: $Y_{1d} = \$40,000$, $Y_{2d} = \$50,000$, $W_1 = \$20,000$ and $r = 5\%$. (NB The graphic is not to scale.) The point on the X-axis corresponds to $C_2 = 0$ and thus $C_1 = W_1 + Y_{1d} + Y_{2d}/(1+r) = 107,600$ as per equation (3). Similarly, with $C_1 = 0$, we have $C_2 = (1+r)(W_1 + Y_{1d}) + Y_{2d} = 113,000$. Point B is defined as a situation with no saving or borrowing ($S_1 = 0$) such that $C_1 = W_1 + Y_{1d}$ and $C_2 = Y_{2d}$.

2.2.1 A present discounted value interpretation of the IBC

The present-discounted value approach makes use of the fact that with an interest rate of r, one is indifferent between receiving \$1 at period 1 or \$(1+r) at period 2. (This assumes no borrowing constraint.) Indeed, a promise to pay \$(1+r) at period 2 allows one to increase consumption today by \$1 through borrowing and is thus equivalent to receiving \$1 today. Conversely, receiving an extra \$1 today allows you to increase consumption by \$(1+r) tomorrow through saving and is thus the same as a promise to give \$(1+r) tomorrow.
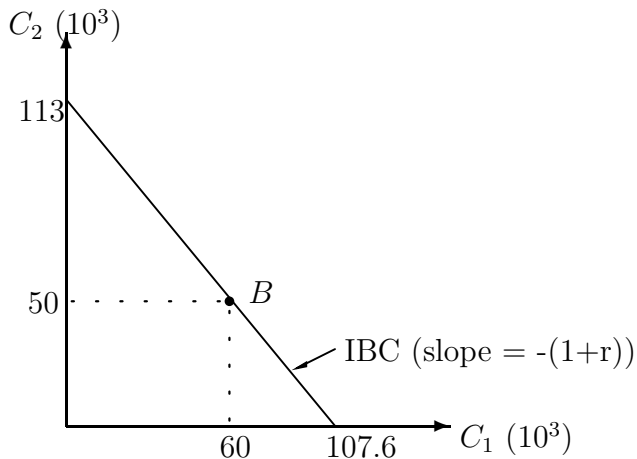
Figure 1: The intertemporal budget constraint

The upshot of the above is that $C_1 + C_2/(1+r)$ denotes the (discounted) present-value (PV) of lifetime consumption while $W_1 + Y_{1d} + Y_{2d}/(1+r)$ is the present-value of lifetime wealth and income, called *lifetime resources*. The IBC can therefore be interpreted as imposing an equality between the PV of consumption and the PV of lifetime resources, i.e.,

$$C_1 + C_2/(1+r) = W_1 + Y_{1d} + Y_{2d}/(1+r). \tag{4}$$

2.3 Consumption preferences

We represent consumer preferences with the help of *indifference curves* between C_1 and C_2. Those indifference curves are assumed to have three fundamental properties:

1. Consumers prefer more to less of either C_1 or C_2. (Welfare increases towards the North-East.)

2. Consumers are ready to trade-off present consumption for future consumption. (Indifference curves are negatively sloped.)

3. Consumers prefer to smooth consumption over time. (Convex indifference curves.)
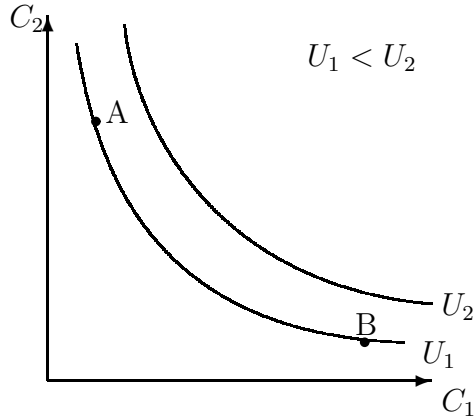
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Figure 2: Intertemporal indifference curves

The slope of an indifference curve denotes the intertemporal *marginal rate of substitution* (MRS) between consumption at each period. Indeed, the slope indicates how many units of period 2 consumption one is willing to give up in order to increase consumption by one unit at period 1. The convexity of the indifference curves implies that the MRS is high when C_1/C_2 is small (point A on U_1), i.e., when consumption at period 2 is high in relation to period 1 consumption, one is willing to give up a lot of period 2 consumption in order to increase period 1 consumption by one unit. Conversely, the MRS is low when C_1/C_2 is large (point B on U_1), i.e., when consumption at period 2 is low in relation to period 1 consumption, one is willing to give up very little of period 2 consumption in order to increase period 1 consumption by one unit.

2.4   The consumer's optimal choice of consumption and saving

The welfare maximizing choice of consumption levels C_1^* and C_2^* is illustrated in figure 3 by point A; it is characterized by the fact that the corresponding indifference curve is tangent to the intertemporal budget constraint. To convince yourself that U_1 is indeed the highest welfare level that can be attained for the given IBC, try any other point on the IBC.

One notes that the welfare maximizing choice of consumption levels has the property that $MRS = -(1 + r)$. Moreover, assuming that point B

corresponds to the no-saving, no-borrowing consumption levels, then figure 3 illustrates a case where the consumer has a positive saving level ($S_1 > 0$).
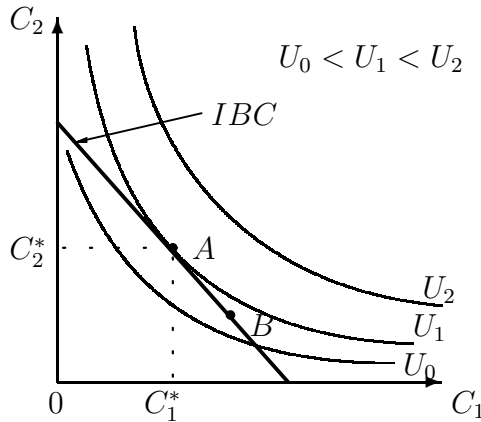

Figure 3: Optimal consumption and saving

One purpose of the above model is to help us understand how consumers decide on their consumption and saving levels. This is already useful in order to explain long-run economic growth. In a later section on the link between expectations and short-run fluctuations, the model will also turn out to be quite helpful in clarifying the long-standing debate over the use of public funds as a means to dampen the effects of a recession. For now, let us see how the model can be used in order to make predictions regarding how exogenous changes in macroeconomic variables such as the interest rate, the wealth levels, or the present and expected income levels, can affect decisions to save and consume.

3   Some predictions of the model

In this section, we conduct *comparative statics experiments*, i.e., we look at how variations in the value of *one* exogenous variable (Y_{1d}, Y_{2d}, r or W_1) will affect *all* the endogenous variables (C_1, C_2 and S_1).

3.1 An increase in present income

Suppose that period 1 income increases by \$5,000 ($\Delta^+ Y_{1d} = 5,000$). One can clearly see in expression (4) that this simply corresponds to an increase in lifetime resources and therefore induces a shift of the budget constraint line to the right from IBC to, say, IBC'. At the new equilibrium point A', we have a case where $\Delta^+ Y_{1d}$ has caused the consumption levels at both periods to increase. This is an instance of *consumption smoothing*: an increase in the income level at *one* period induces the consumer to increase her consumption levels at *both* periods. This can be achieved through the possibility of higher savings.
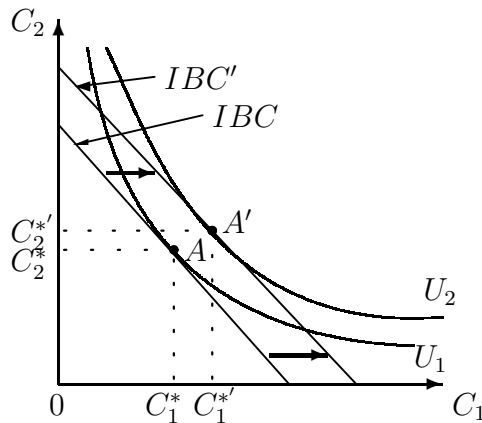

Figure 4: An increase in first-period income

3.2 An increase in the interest rate

We now would like to use our model in order to predict how an increase in the rate of interest affects the decisions to consume and save, say from r to r', with $r' > r$. We look at the case of a consumer whose initial saving level was strictly positive at interest rate r, i.e., the initial equilibrium point A is located to the left of the no-saving, no-borrowing point B on the IBC line (see fig. 5).

With r', the new budget line IBC' must pass through point B also. This is because point B does not depend on the interest rate, as defined in 5.
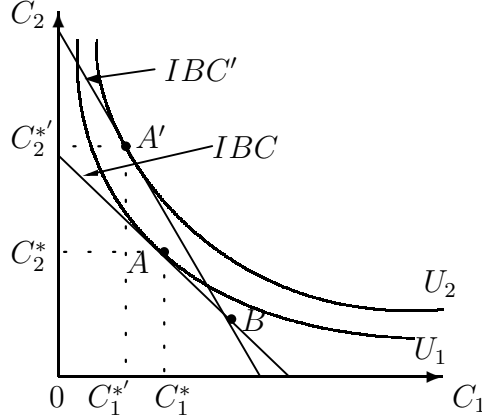
Figure 5: An increase in the interest rate

Since points to the left of B correspond to strictly positive saving, a higher interest rate allows for higher period 2 consumption levels, and conversely for borrowers to the right of point B. With a higher interest rate, savers can therefore increase their consumption levels at both periods through consumption smoothing; this is called the *wealth effect* of a higher interest rate and is analogous to the effect of an increase in period 1 income seen above.

However, with a higher return to savings, the opportunity cost of consumption at period 1 increases, i.e., more of period 2 consumption must be given up for a unit increase in period 1 consumption. This induces consumers to consider a decrease in present consumption in return for more consumption tomorrow; this is called the *substitution effect*.

Consequently, the increase in the interest rate has two opposite effects on period 1 consumption: a substitution effect and a wealth effect. The net effect is *a priori* ambiguous. Which of the two is strongest will depend on the shape of the consumer's indifference curve. Point A' in figure 5 illustrate a case where the substitution effect is stronger than the wealth effect as period 1 consumption drops after an increase in the interest rate. This case is the one that is typically assumed in the macro-economic literature because it corresponds better to the evidence, i.e., in real life, consumers tend to save more when the returns to saving increase.

Exercize 1 *Analyse the effects of the following changes:* $\Delta^+ Y_{d2}$, $\Delta^- r$, $\Delta^+ r$

when $S_1 < 0$, $\Delta^+ W_1$.

4 Borrowing constraint

Often, consumers are not able to borrow money even if they wished to. This may be due to a bad credit history or a lack of collateral (wealth) to use as a guaranty for the lenders. In order to illustrate this situation, let us assume a consumer with zero initial wealth ($W_1 = 0$) as illustrated in figure 6.
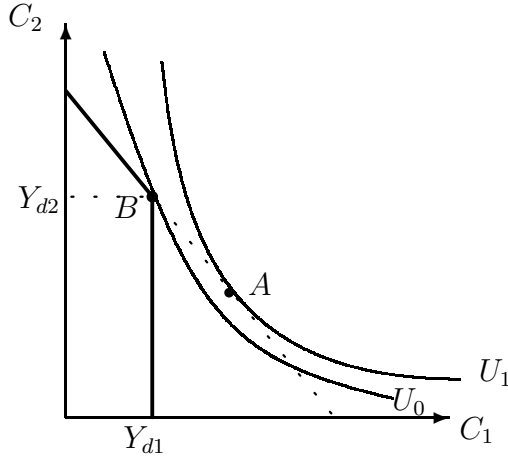

Figure 6: A borrowing-constrained consumer

The consumer's period 1 income level being much lower than period 2's, he would like to smooth consumption between the periods through borrowing and then consume at point A. Since borrowing is not possible, the best that he can do is consume all of his disposable income at period 1 ($C_1^* = Y_{d1}$) and similarly for period 2 ($C_2^* = Y_{d2}$), as illustrated by point B. This leaves him on a lower welfare level than without the borrowing constraint. Note that with the borrowing constraint, it is no more the case that the MRS equals the rate of interest.