MAT3153 Assignment 4

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1. Let X be a path-connected space. Suppose that for any $x_0, x_1 \in X$, any two paths from x_0 to x_1 are path-homotopic. Show that X is simply connected.

2. Let $f : S^1 \to X$ be null homotopic (i.e. homotopic to a constant map). Show that f extends to a continuous map, $g : B^2 \to X$, where $B^2 \subset \mathbb{R}^2$ is the closed unit disk. (Hint: show that B^2 is homeomorphic to a quotient space of $S^1 \times I$.)

3. Let $f : S^1 \to S^1$ be null homotopic. Show that there exist $x, y \in S^1$ such that $f(x) = x$ and $f(y) = -y$.

4. Let $U \subset \mathbb{R}^2$. A *continuous vector field on* U is a continuous map, $v : U \to \mathbb{R}^2$. Let v be a continuous vector field on B^2. Show that there exists $x \in \mathbb{R}^2$ such that $v(x)$ points directly outwards (i.e. $v(x) = tx$ for some $t > 0$).

5. Prove that $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^n$ for $n \geq 2$, and that $\mathbb{R}^2$ is not homeomorphic to $\mathbb{R}^n$ for $n \geq 3$.

6. Let G and H be non-trivial groups. Show that $G * H$ is non-abelian. (Hint: remember that $G * H$ is defined as a set of equivalence classes.)