MAT 2384-Practice Problems on Numerical Methods for Differential Equations

Question 1. For each of the Following IVP’s, apply Euler Method with the given step size \( h \) to estimate solutions on the given interval. Round your answers to 6 decimal places. Then Solve the IVP exactly and compare your estimations with the Exact values.

1. \( y' + 5x^4y^2 = 0, \ y(0) = 1, \ h = 0.1 \) on [0, 0.5]

2. \( y' = \frac{1}{2}\pi \sqrt{1 - y^2}, \ y(0) = 0, \ h = 0.1 \) on [0, 0.3]

3. \( y' = (y + x)^2, \ y(0) = 0, \ h = 0.1 \) on [0, 0.4]

Question 2. For each of the Following IVP’s, apply the Improved Euler Method with the given step size \( h \) to estimate solutions on the given interval. Round your answers to 6 decimal places. Then Solve the IVP exactly and compare your estimations with the Exact values.

1. \( y' = y - y^2 = 0, \ y(0) = 0.5, \ h = 0.1 \) on [0, 0.3]

2. \( y' + 2xy^2 = 0, \ y(0) = 1, \ h = 0.2 \) on [0, 0.6]

3. \( y' = 2(y^2 + 1), \ y(0) = 0, \ h = 0.05 \) on [0, 0.2]

Question 3. Consider the IVP:

\[ y' = 2x^{-1}\sqrt{y - \ln x} + x^{-1}, \ y(1) = 0. \]

(1) Verify that the Exact solution is \( y = (\ln x)^2 + \ln x \)

(2) Use the Improved Euler Method to estimate solutions of the IVP on the interval \( 1 \leq x \leq 1.6 \) using a step size of \( h = 0.2 \). Round your answers to 6 decimal places

(3) Use the Runge-Kutta method of order 4 to estimate solutions of the IVP on the interval \( 1 \leq x \leq 1.6 \) using a step size of \( h = 0.2 \). Round your answers to 6 decimal places
(4) Make a table to compare your estimates in parts (2) and (3) with the exact values (from part (1)).

**Question 4.** Use the **Runge-Kutta method of order 4** to estimate solutions of the IVP

\[ y' = xy + \cos x, \quad y(0) = 0 \]

on the interval \( 0 \leq x \leq 0.6 \) using a step size of \( h = 0.2 \).