The Dynamics of Polarization Mode Dispersion in Field Fibers
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Abstract
The dynamical behavior of the polarization mode dispersion (PMD) in aerial field fibers is reported. A theoretical model is proposed to simulate the corresponding PMD dynamics of an aerial fiber. A PMD emulator based on the theoretical model is constructed and it is found that the emulator can closely mimic the PMD dynamics.

I. INTRODUCTION
Polarization mode dispersion (PMD) degrades the performance of fiber-optic communication systems. The effects of PMD become critical for systems with single-channel bit rates of 10Gbps and higher. PMD of an aerial fiber can result from the broken cylindrical symmetry caused by such effects as bending, twisting, and thermal stress. Because of the dynamic environment, the PMD of an aerial field fiber fluctuates on a very fast time scale. We report in section II the experimental monitoring of an aerial fiber in the winter of 2000. Section III describes the theoretical modeling of the PMD dynamics. Section IV reports construction of a PMD emulator. Section V is the conclusion.

II. EXPERIMENT
A 34-km length of SMF-28 aerial-hoarse tube fiber (with mean PMD of 7.5 ps) cable located at Woodstock in New Brunswick, Canada is monitored for a period of one week in the winter of 2000. Due to the speed limitation of current PMD test equipments the direct monitoring of the dynamic PMD value still remains a challenge. Therefore we decided to monitor the state of polarization (SOP) instead. In the experiment[1] we measured the output (SOP) every 10 ms using a Agilent polarimeter at a wavelength of 1550 nm by launching a continuous light wave at a power of 0 dBm from an Agilent 8164A tunable laser with fixed input SOP. Therefore, as PMD fluctuates the output SOP will change correspondingly. Hence the dynamics of the output SOP will be directly related to the PMD dynamics of the aerial field fiber. We find that the fastest correlation time for the SOP is about 10 ms for the aerial fiber. Similar experiments are done on buried field fibers[2].

III. MODELS
The PMD dynamics of the aerial field fiber is modeled by a highly mode coupled dynamic wave-plate model. In this model we first construct a fiber made of many polarization maintaining fiber (PMF) sections such that the PMD concatenation rules are satisfied statistically, i.e. \(<\tau_{\text{tot}}^2> = \sum_{i=1}^{N}<\tau_i^2>\), N is the total number of wave-plate sections, whose value should be chosen large enough to be in the region of high mode coupling. Here we assume each section’s differential group delay (DGD) \(\tau_i\) is uniformly distributed in \([0, \tau]\) where \(\tau = \sqrt{3(<\tau_{\text{tot}}^2>/N)}\). Once DGD values of each section are chosen they are kept constant in the rest of the dynamic simulation. The mode coupling angle between adjacent PMF sections are used as the models dynamic parameters. This choice stems from the intuitive observation that aerial fibers easily swing in the wind and as such the coupling angle is the most sensitive parameter in the PMD dynamics. This dynamic coupling angle is chosen by using a sequential probability density function (PDF) that has a short term memory. Specifically when all the PMF sections are assumed to have only linear birefringence we choose the following Gaussian PDF[3] to represent their orientation angle \(\theta(t)\):

\[P[\theta(t_0 + \tilde{t})|\theta(t_0)] = \frac{1}{\sigma\sqrt{\pi}} \exp \left\{ \frac{[\theta(t_0 + \tilde{t}) - \theta(t_0)]^2}{\sigma^2} \right\} \tag{1}\]

Here \(\theta_j(t_0)\) is the mode coupling angle for jth PMF section at the time \(t_0\) and the above PDF is used to generate its next coupling angle \(\theta_j(t_0 + \tilde{t})\) at time \(t_0 + \tilde{t}\). \(\sigma\) is a fitting parameter that scales with the experimental measurement time.
interval $\Delta t (= 10 \text{ ms})$. This scaling relationship can easily be seen in the following limit for Eq. (1):

$$
\lim_{\sigma \to 0} P[\theta_j(t_0 + \Delta t) | \theta_j(t_0)] = \delta[\theta_j(t_0 + \Delta t) - \theta_j(t_0)] \frac{1}{|\theta_j(t_0)|} \delta(\Delta t)
$$

(2)

In the model we treat a complete update for each PMF section's coupling angle as a measurement time interval. Following is a comparison of the SOP angular correlation density (SOPACD) at time interval of 10 ms between the experiment and simulation. Where SOPACD at time interval $\tilde{t}$ is defined as the distribution of the angle between Stokes vectors separated by time interval $\tilde{t}$. More specifically, the angle between the normalized Stokes vectors $\tilde{S}_{t_0}$ at time $t_0$ and $\tilde{S}_{t_0 + \tilde{t}}$ at time $t_0 + \tilde{t}$ is calculated as:

$$
\gamma(t_0, \tilde{t}) = \arccos(\tilde{S}_{t_0} \cdot \tilde{S}_{t_0 + \tilde{t}})
$$

(3)

As one varies the time $t_0$ in the Eq. (3) one can get a distribution of the SOPACD for time interval $\tilde{t}$. Figure 1(a) shows a fit of the model and the experimental field fiber at time interval $\tilde{t} = 0.1$ ms, Figure 1(b) shows the same at time interval $\tilde{t} = 100$ seconds.

![Figure 1](image)

**Figure 1** Experimental with 6.2 million points (solid) and theoretical with 3.2 million points (ragged) SOPACD histograms. 1000 bins and (a) $\tilde{t} = 0.1$ and (b) $\tilde{t} = 100$ seconds correlation time. (aerial fiber)

In our model simulation we have treated a complete model update of the coupling angles $\theta_j$ (i.e. each PMF fiber section has had a update to choose their new orientation angle according to Eq. (1)) as corresponding to a time interval $\tilde{t}$ in the experiment for aerial field fiber. In such a simulation our model parameter is found to be $\sigma = 0.019 \text{ rad}$ in 1(a) and $\sigma = 0.110 \text{ rad}$ in 1(b), respectively, when one uses $N = 50$. To further probe the relationship between our model parameters $\sigma$, the number of fiber sections $N$, and the correlation time interval $\tilde{t}$ we extensively analyzed the simulation and the experimental results finding the following:
\[ \sigma(\tilde{r},N) = \kappa \sqrt{\frac{\tilde{r}}{N}} \]  

(4)

where \( \kappa \) is a constant parameter related to the environmental impacts to the field fibers. We found that for the aerial field fiber \( \kappa = 7.735 \times 10^{-2} \text{ (rad/\sqrt{second})} \). It is interesting to note that \( \kappa^2 \) has the dimension of angular diffusion coefficient. Hence in essence our proposed dynamic model is rooted in the theory of random walks. It is obvious that one can use the \( \kappa \) parameter to characterize the PMD dynamics of a field fiber. In general the larger the \( \kappa \) value the faster the SOP fluctuates.

IV. EMULATOR

Having seen such a good agreement between the simulation and the experiment in the last section we decided to build a dynamic PMD emulator that closely mimic the aerial field fibers. However despite its simplicity the construction based on the model in the last section turns out to be more challenging in practice. Therefore we instead constructed an emulator using PMF interleaved with polarization controllers (PC). Each of the PCs is made of four fiber squeezers and their relative spatial orientations are fixed at \((0^\circ, +45^\circ, -45^\circ, 0^\circ)\). However the amount of squeeze can be tuned within a finite range such that the birefringent phase delay for a wavelength under consideration is in the range \((0, 2\pi)\). To model the dynamics of the aerial fiber we introduce a similar sequential PDF for each fiber squeezer

\[ P[\tau_j(t_0 + \tilde{\tau}) | \tau_j(t_0)] = \frac{1}{\beta \sqrt{\pi}} \exp \left\{ -\frac{[\tau_j(t_0 + \tilde{\tau}) - \tau_j(t_0)]^2}{\beta^2} \right\} \]  

(5)

subjecting to periodic boundary condition \( \tau_j = \text{mod}(\tau_j, \tau_{2n}) \). Where \( \tau_j(t_0) \) is the amount of squeeze induced differential group delay (DGD) for jth squeezer at time \( t_0 \), and \( \tau_j(t_0 + \tilde{\tau}) \) is the amount of squeeze induced DGD at time \( t_0 + \tilde{\tau} \). Furthermore \( \tau_{2n} \) is the squeeze induced DGD that results in a \( 2\pi \) phase delay, and \( \beta \) is a parameter that represents the PMD dynamics. The emulator is made of five PCs and five PMFs having DGD values of 10, 5, 1, 0.89

Figure 2 The SOPACD comparison of the emulator output with a theoretical model output.
and 0.46 ps respectively. Those PMFs also have a small PDL values corresponding to 1.63, 0.16, 0.15, 0.54 and 0.53 dB respectively. Same as our theoretical model discussed in the last section, we again treat a complete update of all the squeezer as an experimental measurement interval. A close agreement between the emulator's output SOPACD and the theoretical model fit is seen in Figure 2.

V. CONCLUSION
We have reported a long term SOP experimental measurement for an aerial field fiber in the winter of 2000. From this experimental result we have proposed a dynamic highly mode coupled wave-plate model to mimic the PMD dynamics. We found that it is critical to employ a PDF that can capture the short term memory effect of the field fiber. Our work sets a new standard[4] of realistic PMD emulation for field aerial fibers.

REFERENCES