CHAPTER 6

TIME, NORMS, AND STRUCTURE IN NINETEENTH-CENTURY PHILOSOPHY OF SCIENCE

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Und mit starren Fingern
dreht er, was er kann.

–Wilhelm Müller[1]

Introduction

Analytic philosophy is generally described as an Anglo-American movement, even though, with the exception of Russell, Moore, and Ramsey, most of the early figures were from Germany or Austria. Furthermore, the problems these philosophers dealt with did not emerge from the head of Zeus. They were the product of a long development over the nineteenth century. Finally, since early analytic philosophy is essentially philosophy of science and mathematics, that development took place only partially within philosophy departments. Many of the early analyticians' heroes were working scientists such as Helmholtz, Heinrich Hertz, Poincaré, and Einstein. What we know about these figures is usually filtered through the interpretations of their work given by their philosophical successors, for instance the Paul Hertz/Moritz Schlick edition of Helmholtz's epistemological writings (Helmholtz 1921, 1977).

[1] In memory of Catherine Liepins née Veiland, Kiev 1899–Toronto 1994 and Herbert Edward Liepins, Riga 1898–Toronto 1994.

But these interpretations are highly coloured, and it is fair to say that historians of philosophy have followed a pattern, beginning with Russell himself, of skipping the nineteenth century. Kant is seen as the last philosopher of note, and from him one jumps straight to Frege, perhaps with a glance at Helmholtz and Poincaré along the way. Doctrines from the late nineteenth century are often retrojected onto Kant, with the result that this century cannot happen, since all essential moves were made before it began. This chapter is an attempt to counter this long-standing approach, so that we may better appreciate the problems that actually confronted turn-of-the-century thinkers such as Frege, Russell, Wittgenstein, Reichenbach, and Carnap. The goal is therefore to restore our depth of field.

I will focus on the antecedents to a variety of approaches typical of twentieth- and twenty-first-century analytic philosophy, such as *formal logic*, *logical empiricism*, *conventionalism*, and the *semantic view*, as well as theories that invoke notions of *normativity*. All of these respond to Kant's philosophy of science; however, it is not my goal to show how they are present *in nuce* in Kant. On the contrary, I will argue they are all solutions to problems posed by his philosophy, problems which he caused as opposed to solving.

The paper has four main sections, devoted to Kant, Helmholtz, Hertz, and early analytic philosophy. I track the evolution of two 'deductive' strategies, one based on the structure of manifolds of representation, one which appeals to regulative conditions on objective experience. The two strategies are connected through a single principle of natural science, namely the law of inertia, which connects uniform rectilinear motion to force. *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.* Different parts of the law correspond to different elements of Kant's *Critique*. The materials for the definition of uniform motion in a right line are given in the Transcendental Aesthetic (TA), whereas the dynamic element derives from the category of causality, whose applicability to experience must be given a different sort of justification. All the scientists and philosophers I discuss in this chapter were concerned with the status of this law, which they generally took to follow from the principle of sufficient reason. I should warn at the outset that, should one take the law to be a *definition*, one is standing at the terminus of the movement I am describing, and not at the point of departure (on the evolving epistemological status of the law, see Hanson 1963 and, above all, DiSalle 2006).

Because this is history of long duration, ranging from works of Euler in the mid-eighteenth century to retrospective assessments by Einstein at the beginning of the twentieth, I cannot offer detailed systematic and textual support for all the connections I draw. Thus I must ask for the reader's forbearance in cases where my treatment is too schematic. I have tried, whenever possible, to provide references to sources which provide some of the missing details. The scope necessarily means neglecting certain authors, most obviously Mach, Boltzmann, Poincaré, Frege, and Russell, but such omissions do not mean that I believe their work to be unimportant, merely that I lacked the space and time to deal with it here.

6.1 Kant

6.1.1 The Structural and Normative Deductions

Kant had two distinct approaches to the problem of demonstrating the validity of foundational principles in the sciences, for the *Critique of Pure Reason* contains *two* transcendental deductions, one for the pure concepts of the understanding (the categories), and one for those of space and time.[2] When we speak of the Transcendental Deductions, we typically mean the A- and B-versions of the deduction of the categories, but in its introductory sections Kant remarks that the need for a deduction is already familiar to his reader, for 'we have above traced the concepts of space and time back to their sources, as well as determining and explaining their objective validity *a priori*, by means of a transcendental deduction' (B119–20). I call this other transcendental deduction the Spatio-temporal Deduction (SD), in order to avoid any confusion with the Transcendental Deduction(s) (TDs) in the standard sense just explained. The Spatio-temporal Deduction has the same purpose as the Transcendental Deduction proper, but it achieves its goal by grounding a priori cognitions in structure, and is thus in a sense superior to the latter. In particular, it can satisfactorily answer the following question, whereas the Transcendental Deduction itself cannot: What guarantee have we that the a priori principles derived from the concepts being transcendentally deduced will be valid under all circumstances, that is to say at all places and times?[3]

This suggestion would seem to be confirmed by Kant's amplification that metaphysical concepts, such as substance and cause, pose a special problem when compared to the concepts of space and time. A transcendental deduction is supposed to explain how concepts can relate to objects a priori (B117), and here there is a dichotomy between 'mathematical' and 'regulative' concepts, which could be glossed in a number of ways, but which I will explain in terms of their relation to time. The concept of a cause, for instance (B122f.), involves a relation of necessitation between two states, but Kant allows that appearances 'can be given' which, while in conformity with the formal conditions imposed by space and time, nevertheless cannot be subsumed under a causal relation. So whereas the concepts of space and time are transcendentally deduced by grounding their validity in the *form* of all appearances, thereby ensuring that every presentation of an object will conform to these concepts (B125f.), such a deduction is not available in the case of (at least some of) the categories, because appearances 'can be given' which do not do so.

[2] See Merritt 2009.

[3] This question and its relation to Helmholtz, Hertz, and the picture-theory have recently been discussed in Patton 2009, which provides a useful overview of much of the literature. Patton's article deals with what she calls the problem of validity or *Gültigkeit*. 'The problem is a variant of the problem of induction, since establishing the validity of a principle of inference, based on regularity, involves making an inference beyond objects as experienced' (Patton 2009, p. 282).

Given that the result of the deduction of the categories will be that, in some sense, appearances not conforming to the categories *cannot* be given, it is helpful to understand in what sense they could. It is this: at least those categories which Kant terms *dynamic*—for instance the traditional categories of causality and substance—refer to relations between realities, that is to say bits of matter or 'beings in time' (B182). Such realities 'fill' time (as well as space), which means for Kant that they can mark definite locations and thereby *determine* time and space by setting limits to (terminating) regions of these otherwise empty 'continuous *quanta*'. But there is nothing in the naked form of intuition to preordain the distributions of beings in time and space. In this sense, appearances could be given that would not conform to the dynamic categories. Because categories such as causality or substance apply to patterns amongst spatio-temporal distributions of beings, the problem of showing the necessary validity of the dynamic categories is at heart the problem of showing how a temporal sequence can be shown to conform to a pattern *necessarily*, without making the pattern part of the structure of representation itself.

6.1.2 Experience

Kant's *Critique* is somehow concerned with what he calls 'conditions on possible of experience'. Much disagreement hinges on how the term experience should be understood. Was Kant talking about our mental life or about our scientific practice, and was he perhaps fallaciously arguing from facts concerning to the first to normative conclusions concerning the second? It is therefore useful to recall that the word experience has a participial form, *experiment*, which Kant's contemporaries sometimes call a 'prepared experience'. Prepared experiences differ from everyday experiences in a number of respects. First, experiments are repeated under varied conditions, with the aim of isolating the necessary and sufficient conditions for changing observables. Second, they often make use of mathematics, in particular algebraic equations, to describe these observables. The causal dependencies we identify can then be expressed algebraically as equations of condition. In consequence, before even considering the physical theories such experiments may support or refute, certain basic conditions on the objectivity of experiments must be met, without which they cannot be reproduced at other times and places. In today's language, we would say that our experimental activity rests on a suite of *metrological sciences*, which lay down norms of measurement. They do so in part by specifying classes of physical object that, using the terminology of contemporary metrological science, *realize* the ideal *representations* of *definitions* of units of measure. For instance, the physical lasers in standards laboratories realize imagined lasers, designed on paper to represent the definition of an SI-metre in terms of a relation between physical light and the second.

For Kant and his contemporaries, there are two fundamental a priori metrological sciences, namely *geometry* and *chronometry*. Without these sciences and the norms they ground, it would be impossible for scientists working at different times and places to swap and check each other's data. In this sense, these sciences are conditions on the objectivity of experimental data, on possible prepared experiences, and conditions for the formulation of algebraic laws. Because they must mediate between timeless formal objects and changing material bodies, metrological objects have a foot in both worlds. As a rule, they imperfectly realize the very quantities they represent, meaning that they must ultimately be normed with respect to an ideal, the first of which was Pythagoras' theorem for the *norma*, or set-square. This dual nature explains the repeated occurrence of metrological examples in the literature from Plato to Kripke.

It is this original sense of the term *norm* that underlies the much-discussed normative aspects of Kant's philosophy. It is only against the background of geometry and chronometry that scientific laws can be stated, confirmed, or refuted, and in this sense they are conditions for the possibility of scientific experiences, as well as conditions for the possibility of formulating causal laws mathematically. At the same time, however, the explanation and isolation of such principles are basic conditions for an open-ended project, that is to say a complete or 'architectonic' description of the natural world.

6.1.3 Geometry, Chronometry, and Phoronomy

While it is commonly observed that the term *geometry* reflects the Greek origins of this science in land-surveying, it is worth remembering that geometry did not become fully independent of this role until the end of the nineteenth century, when it was finally brought into the fold of pure analysis. For eighteenth-century scientists, geometry is a science for measuring bodies, and it is still extraordinary because it makes true empirical claims even though it is based on apparently a priori axioms. To take one prominent example, Gauss's research into the problem of mapping spherical relations onto Euclidean ones—the same research Riemann used as his point of departure—was directly connected to cartographic questions. And one of the main preoccupations of the Swiss mathematician and philosopher of science Johann Lambert, to whom Kant considered dedicating the *Critique*, concerned map projections, what we would today call projective geometry. In other words, while geometry plays an essential role in physics (the study of motion), this was far from being its only natural scientific application.

What is less well known is that since the early modern period a similar a priori science existed for measuring time. In contrast to geometry, however, the a priori principles at the basis of chronometry are exceedingly simple. It presupposes only a few basic 'axioms', such as that time has one dimension, that it is continuous, that its parts are not simultaneous, etc. (Lambert 1771; Schultz 1789). Taken together, chronometry and geometry give rise to a third fundamental a priori science, namely 'phoronomy' or kinematics, which articulates laws for the composition of speeds and accelerations, such as the parallelogram law, and thereby makes the quantification of motion possible.

Those familiar with Kant will easily recognize that each of these sciences has a direct counterpart in his work, and that each is given its own separate foundation or 'transcendental deduction'. The possibility of the sciences of geometry and chronometry is explained in the Transcendental Aesthetic's sections on time and space, but here Kant observes in conclusion that he has also thereby grounded the a priori cognition displayed by 'the general doctrine of motion', that is to say the Phoronomy of his *Metaphysical Foundations of Natural Science* (*MFNS*). The latter science is *empirical* in the sense that it concerns matter and time; however, it is *pure* because it depends only on the structure of pure intuition. This pure kinematics will be central to our subsequent discussions of both Hertz and Helmholtz. In order to get at the connection, we need to consider briefly the status of this science in eighteenth-century physics and mathematics.

It is generally accepted that Kant abandoned continental relationalism at least in part because of arguments advanced by the mathematician Leonhard Euler, in particular his 'Reflections on Space and Time', where Euler sided with Newton against Leibniz and his rationalist followers such as Wolff.[4] Euler had argued that the law of inertia proved the existence of independently existing parts of space and time, which sustained intrinsic relations of equality and simultaneity. Since the law is incontrovertibly *true*, Euler argued, bodies must be 'regulating' their behaviour relative to something real. Thus we must be absolutists not only concerning the existence of space and time, but indeed concerning their metric.[5] The independent existence of such structures is forced on us by the law, which Euler, like many of his contemporaries, regarded as the fundamental principle of natural science.

Thus in saying that the Transcendental Aesthetic provides a transcendental deduction of the concepts of space and time, Kant follows Euler in arguing that the possibility of geometry, chronometry, and kinematics presumes the existence of these same

[4] See Janiak 2009 on this question.

[5] Euler actually has *two* laws of inertia, one for rest and one for uniform motion; however this distinction can be ignored for our present purposes. It is sometimes suggested that Euler was arguing the opposite, and that Kant followed him on this score. 'Kant there appears to follow §§ 20, 21 of Euler's paper in rejecting the idea that the equality of times can be rendered intelligible independently of the law of inertia. Instead, Kant appears to hold that the law of inertia *defines* the equality of times: equal times are those during which an inertially moving body traverses equal distances' (Friedman 1992, p. 20n30). In these sections, Euler does indeed call the equality of times into doubt, but not to suggest that it be defined in terms of the law, rather as the premise of a reductio: '... if time is nothing other than the order of successions, then how will one make the equality of times intelligible? ... As this equality could not be explained by the order of successions, just as little as the equality of spaces can be so by the order of coexistants, and since it enters essentially into our principle of motion, one cannot say that bodies, when pursuing their motion, obey something that exists only in our imagination' (Euler 1750, pp. 332–3). That is to say, without assuming an independent metric of both space and time, conformity of motions to the law of inertia would be inexplicable. See the passage from B67 discussed below in the section on Helmholtz's theory of space as well as Timerding 1919 and Jammer 2006, pp. 82–3.

The reason for this misunderstanding is perhaps a series of errors in the standard English translation, e.g. 'Since this equality cannot be explained by the order of succession, so just as little can the equality of spaces be explained by the order of co-existent ones, and whether it enters essentially into the principle of motion ...' (Euler 1967, pp. 125–6). A parallel, apparently deliberate, misreading is made in § 5: 'Thus it is certain, that if it is not possible to conceive the two principles adduced from mechanics without their being involved in the ideas of space and time ...' (Euler 1967, p. 118), versus 'Il est donc certain, que s'il n'etoit pas possible de concevoir les deux principes allegués de la Mecanique, sans y mêler les idées de l'espace & du tems [*sic*] ...' (Euler 1750, p. 326).

structures. For Kant, however, they must be inherent to human sensibility, for we could not otherwise explain our a priori knowledge of them. The Transcendental Aesthetic 'explains the possibility of as much synthetic knowledge a priori as the general theory of motion evinces' (B49), because it provides the material for the definition and laws of uniform motion found in the Phoronomy of the *Metaphysical Foundations*. Since deviation from uniform motion counts as a state change, thus as something requiring a sufficient reason, the application of the a priori law of causality to these definitions results in the principle that any deviation from uniform motion has a sufficient reason, in other words in an a priori derivation of the law of inertia (*MFNS*, 4.541–2).

Nevertheless, the foundation provided for all three sciences is provisional, for Kant has not yet explained the possibility of *quantifying* space, time, and motion. All three sciences must also be arithmetized if we are to formulate laws of nature in terms of the pure concepts of quantity, that is to say by means of algebraic equations of condition.

6.1.4 Algebra, Quantification, and Change

As recent work by Longuenesse (1993, 2001) and Shabel (1998) suggests, we find a clean split in Kant's theory of mathematics between classical Euclidean geometry and the new analytic geometry typical of Galilean–Cartesian mechanism. The quantitative categories (unity, plurality, totality) allow us to think 'the concept of many in one', that is to say to conceive of phenomena as denumerated sets. I suggest that for Kant and his contemporaries, this is part of a larger project, which I will term the *quantification of nature*. The mathematicians Kant knew best—Lambert, Euler, Kästner—see it as the aim of natural science to describe physical change first by means of numbers, then in terms of 'equations of variable quantities', which can increase or decrease in time. In the *Critique*, this is reflected in Kant's claim that all pure concepts attach to experience by means of a temporal 'schema', where, in the case of the categories of logical quantity, this schema is *number.*

Just as geometry must be quantified in order to become analytic geometry, the same must be done for chronometry and phoronomy. The passage of time, the quantities of motions, the intensities of forces, and, eventually, quantities such as temperatures and pressures, must all be open to arithmetization if an algebraic science of nature is to be possible. This possibility is explained in the Axioms of Intuition and the Anticipations of Perceptions of the *Critique*, which argue that all perceptions are implicitly quantifiable because all perception occurs in time (B201f., 23.28). The Phoronomy of the *Metaphysical Foundations* confirms this hypothesis, since Kant's main concern there is to ground the additive properties of motions (the parallelogram law for velocities). But he does so under a peculiar constraint: the motions being added must be represented as occurring within a single unit of time, for otherwise the component motions would not literally be parts of their sum. In other words, Kant is giving an account of the additive properties of velocity-differentials, which correspond to what he calls 'intensive quantities', namely ones which do not contain parts, but can be quantified with respect to their increase or decrease in time. Once we have an account of these additive properties, we can describe arbitrary motions by synthesizing, that is to say integrating, these differentials in time.

6.1.5 Causality

As we have seen, the fundamental metrological sciences are grounded in our intuitive knowledge of the structure of time and space asserted in the Transcendental Aesthetic and extended in the Phoronomy of the *Metaphysical Foundations*. This intuitive knowledge of space gives rise to a priori propositions concerning lines and figures, for instance that two lines cannot enclose a space (B65, B204), which can only be synthetic because they do not follow from the law of non-contradiction (B268). When augmented by the temporal schema of quantity, namely number, it grounds analytic geometry. In all cases, the principles in question must rest on what Kant calls 'constructions' of the requisite objects in intuition, since they are not analytic.

But there are no corresponding objects in the case of time, because its parts are never simultaneously given and therefore cannot be compared. Memory may give an impression of elapsed time, but memories cannot provide a basis for the quantification of time. This presupposes cyclical or oscillatory motions such as those of heavenly bodies or, in the lab, chronometers. Because these are taken to be non-uniform, they count as alterations subsumable under the concept of causation. So the doctrines of the Aesthetic, which ground the fundamental metrical characteristics of space, time, and motion, only achieve their completion when conditions on time-measurement have been given. For this reason, the doctrine of time set out in the Transcendental Aesthetic is completed in those long sections of the *Critique* concerned with 'time-determination' (the Schematism, the Analogies). In Kantian terminology, we would say that although the continuity, dimensionality, and ordering relations of time have already been grounded, its three *modes*—perdurance, succession, and simultaneity—can only be grasped by means of the relational categories, which connect the parts of time sequentially through necessary causal connection, and 'ubiquitously' when objects at different places are bound together by reciprocal causal relations (for instance, gravitational or electrostatic forces), and can therefore be regarded as simultaneous or not.

Kant inverts the analytic sequence in his typical fashion. From the empirical point of view, the claim is that objective time-measurement can only occur when we possess physical objects such as clocks, whose laws of oscillation or 'continuous alteration' are known. For it is, Kant argues, only rotational, accelerated motion that is actual or real (*wirklich*, 4.142ff.). Transcendentally, the argument is that without applying the dynamic categories of substance, cause, and community (reciprocal action), we would have no way of drawing a division between subjective and objective time-flow. Whereas, as Kant's contemporary Lambert observes in 1771, in a sequence of elements ordered by a necessary and asymmetric logical relation, the elements are connected to one another 'like the wheels in a clock', and can serve to norm time.

Kant's argument is straightforward. Recall that the mere ordering relations posited in the Aesthetic do not ascribe to time either direction or flow, which involves the supplementary 'mode' of succession and its corresponding category, namely causation. By contrast, our experience of time as a succession of moments reveals an implicit and pre-conceptual modal structure (23.22), for each *momentum* entails its successor, a necessity we sometimes express by saying that we cannot go back in time, whereas we can go back in space. But even within the succession of time, a difference between coupled states that resist my will and those that do not can be discerned. Certain kinds of experiential successions are reversible, whereas others are not. I can choose the order in which I examine the various parts of a house, but not the order in which I experience the motion of a ship following the flow of a river (B238). I call this resistance to my will exhibited by certain successive phenomena an objective cause. It is with respect to such indefeasible regular connections that I norm the passage of objective time.

The cognitive relation of logical consequence must, therefore, be given a corresponding 'figurative' representation, a structure in pure sensibility that maps the intellectual concept of causation onto the material world. (Cf. B152, 7.191.) For, as Kant insists, there can be no formal or qualitative similarity between the categories and the spatio-temporal appearances they might represent. If there were such a connection, the possible (if problematic) applicability of the categories to non-spatial temporal appearances would be called into doubt, undermining a fundamental aim of the *Critique*. Each category has two such figurations: first a figuration in time, which Kant calls a *schema*, second a figuration in both space and time, which, in the case of the dynamic categories, he calls *analogies*. For instance, the logical concept if-then expresses the idea that when one thing is posited, another thing is posited of necessity. The temporal *figure* of causation is that of a part of time (a moment) that necessitates its successor, and its spatio-temporal figure is that of a pattern of material appearances that are subject to a universal rule (cf. the schemata of causality and necessity at B184–5). In projecting the first sort of figure onto space and matter to produce the second, we 'legislate to nature' (B160).

6.1.6 The Transcendental Deductions

With these remarks in place, I will now turn to a brief discussion of the Transcendental Deduction proper, considering only its role in grounding a priori principles in the sciences. My claim will be that, precisely because there is no formal similarity between the categories and their subsumpta, there is also no obstacle to making categorical subsumptions, thus that we are justified in subsuming categorically to the extent that doing so is a condition for our grasping, or apprehending, the manifold of appearances.[6] We may do so even in the face of contrary evidence, for everyday phenomena constantly violate the law of causality.

The form of Kant's argument faithfully reflects his own characterization, namely that it is an 'original acquisition' of the *right* to employ the categories, which defers the

[6] See Anderson 2001, pp. 291f.

question of their alethic validity in favour of a deontic grounding (Henrich 1989). The acquisition is justified as a condition of the Cartesian *cogito*, in the same way that an individual's right to unowned property in the state of nature is created *ex nihilo* through the act of seizure itself, while it is *justified* as a condition of his survival, a so-called 'right of necessity'. The Deduction describes how I would acquire such a right were I to think categorically, by appropriating the external and making it *mine* (6.258, B131f.).

This form of deduction can be applied psychologically or science-theoretically. One could take Kant to be arguing the case of a noumenal subject, who spontaneously applies the categories to experience in securing his existence through time. Or one can read him as laying down conditions of objectivity, which must be observed in order to produce a unified account of nature. In either case, the categories are justified relative to an end—consciousness or, in Helmholtz's later version, the 'complete comprehensibility of nature'—but that end is not really a matter of deliberative choice. As Wittgenstein later put it, we follow the rules deriving from the categories blindly, but not without justification.

Kant contrasts this deductive method with two others. First, a 'despotic' or 'dogmatic' deduction such as is found both in traditional and Wolffian metaphysics, which grounds a priori truth through an appeal to authority, and may indeed presume insight into the mind of God himself. Second, a 'physiological' deduction such as provided by British empiricists, who explain a priori principles in terms of innate, hardwired dispositions, but thereby lose the ability to explain why such principles are binding on physical reality. Kant avoids the problem entirely with a promissory note. The law of causality provides nothing more than the germ of the completed tree of nature, by which I mean it provides principles on whose basis natural science *should* be constructed, but which will be alethically valid only once natural science is complete. Universalizing the category of causation across time and space produces a causal law that can thus be asserted, with justification, in the face of recalcitrant evidence.

The 'non-homogeneity' of the categories with experience—the fact that they have no formal similarity to phenomena—plays a dual role here. First, because the categories are not and cannot be *in* the data they subsume, we must explain and justify their applicability to experience. Such a justification would be straightforward in the case of an empirical concept such as *plate*, for I need appeal to nothing beyond the true statement 'I once saw a plate' to justify my talking about other plates, whether actual or possible. Furthermore, because the concept of a plate contains that of roundness, and roundness is a property of spatial forms, this concept is 'homogeneous' with the intuitions it can subsume (B176), which in turn means there can be a question of right or wrong subsumptions. But Hume made such a justification of causal concepts impossible, and Kant accepts his result: no finite series of observations ever contains the necessary connection which is a cause, meaning that no one ever has seen nor ever will see a cause. There will never be an experience that justifies my use of causal concepts in the way that my concept *plate* was justified.

Kant turns this difficulty to his advantage. Clearly I can be mistaken in any causal judgement, such as *the sun warms the stone* (*Prolegomena* 4.301), but this singular judgement is neither verified nor refuted by this single (positive) case. If I subsequently judge that the sun did *not* warm the stone, this will be because of other observations of other objects placed in the sun, not because I discovered that what I thought was present in the first warming event (Hume's necessary connection) was actually absent. The truth-conditions of this judgement are in other words intrinsically linked to the possible outcomes of other experiences or experiments elsewhere in space and time. Similar remarks apply to the relation between numerical concepts and the spatio-temporal objects they may subsume.

In both cases, the category in question relates to the phenomena only through the medium of time. Although nothing in sensory experience corresponds to a cause, the concept of a cause can be represented as the figure, 'moment which necessitates a subsequent moment in time', that is to say by ascribing a structure of directed entailment to time itself. Similarly, since every physical object can be partitioned in many ways, there is no direct relation of correspondence between numerical concepts and physical objects; rather these are mediated by the temporal schema of the logical quantities and forms of judgement (all, some, one). This is 'a representation that comprises the successive addition of one to one (of the same kind)' (B183), that is to say the concept '*one* set [*Menge*, B112] with *some* (*n*) members of the set of *all* these things', where the value *n* is the result of an ideal measurement operation.

Kant makes this doctrine explicit at two points in the *Critique*, namely when he introduces the notion of a 'figurative synthesis' in the Transcendental Deduction, and when in the next chapter, the Schematism, he insists that there is no shared form between categories and sensibility. *Figurative representations* depict one kind of thing by means of things of another kind, as notes on a scale depict sounds. An extreme case of figuration is when the intellectual is represented by means of the sensible, where the figures are there only as a means for representing concepts (7.191, on the *facultas signatrix*). So a figurative representation can represent in the absence of any formal or qualitative resemblance, for instance as is done in algebra and symbolic logic.

When Kant says that the schemata provide the missing link between category and sensation, he means that the schemata are figurative representations of the categories, produced, as he explains, by an 'influence of the understanding on inner sense'. But these figures are not figures in space, they are figures in time. Kant calls such a purely temporal synthesis of possible experiential data the *transcendental synthesis of the imagination*, distinguishing it sharply from a subordinate *pure sensible synthesis* which generates forms in space, and which is usually identified by commentators with the transcendental temporal synthesis on which it depends.[7]

[7] B151f., B185, above all the list at 23.18–19, 'The productive imagination is 1. empirical in apprehension 2. pure but sensible with regard to an object of pure sensible intuition 3. transcendental with regard to an object in general. The first presupposes the second and the second the third.

The pure synthesis of the imagination is the ground of the possibility of the empirical one in apprehension thus also of perception. It is possible a priori and produces nothing but shapes. The transcendental synthesis of the imagination concerns only the unity of apperception in the synthesis of the manifold in general through imagination. Through this the concept of an object in general is thought according to the various types of transcendental synthesis. The synthesis occurs in time . . .'

In conclusion, we can see how the deductive grounding of the dynamic categories differs from that of the mathematical ones, and from the deduction of the concepts of space and time. Concepts and principles that concern the form of spatial intuition itself—line, area, contain—are based on direct intuition and construction of their objects by the pure sensible imagination, objects which give these concepts a 'sense and reference' (*Sinn und Bedeutung*, Letter to Tieftrunk 13.224, A240/B299). Similar constructions for motion are advanced in the Phoronomy of the *Metaphysical Foundations*. However, quantifying space, time, and motion means applying both the quantitative categories (all, some, one) and their correlated schema (number) to these constructions. Line segments are produced by the imagination through successive aggregation, or synthesis of their homogeneous parts, just as we imagine the successive addition of the parts of time. Both are *extensive* quantities that literally contain their parts (B202f.). In the case of an *intensive*, differential concept such as speed, quantification involves ordering the intuitions according to their growth from zero to an arbitrary value (B207f., *MFNS* 4.490f.). We get analytic geometry in the case of space, and differential analytic kinematics in the case of motion. Theorems such as Pythagoras', which involve applying quantities to geometrical objects, are verified by the constructions to which they refer, and without these constructions, there would be no way of proving such non-analytic principles. For the concept of a triangle is equally applicable to objects on the surface of a sphere, and yet these have quite distinct characteristics, which were well understood in the spherical trigonometry of Kant's day (4.285).

By contrast, the deduction of the dynamic category of causation does not produce a principle that is verified by the forms of space and time, which is why Kant says that it is merely regulative, and not constitutive. When we consider the naked structure of time as laid out in the TA, we have only ordering relations, no directional asymmetry, and no notion of necessary succession. The 'determination' of time by the category of causality describes the structure of directed necessitation that Kant calls 'succession' in terms of pure logic. The events that fill each part of time should necessitate their successors, as premisses do their consequences. If the simultaneous parts of space that correspond to a moment in time are conceived as subject to a rule that necessitates the next moment, then this rule will be deterministic. The states it regulates can be nothing other than distributions of material properties (intensive quantities) in the manifolds of space and time. But the law of causality that dictates this modal structure—every temporal event has a sufficient reason that precedes it—is at most a demand we place on phenomena, one that everyday phenomena violate every day.

While we 'legislate to nature' (B159, Schultz 1785, p. 299) by imposing this normative demand on the physical world, we do so with the knowledge that the laws will be violated. Only when the project of a complete science of nature is complete will we be in possession of a conceptual system in which this law is true. So the *Critique of Pure Reason* and its physical extension, *The Metaphysical Foundations of Natural Science*, lay the cornerstones of a project that extends through the nineteenth century, namely the construction of a system of rational mechanics that will allow us to acquire or apprehend all natural phenomena within a closed causal system. Such a demand could be fulfilled, Kant and his contemporaries believed, by a mechanics of mass-points subject to central forces (Hyder 2009). For in such a mechanics, the sufficient reason of the future motions of a system is the current spatial disposition of its mass-points. Kant's successor, the physicist and physiologist Hermann von Helmholtz, called this regulative ideal the 'complete comprehensibility of nature'.

6.2 Helmholtz

Now that we have outlined the two distinct deductive strategies used by Kant to justify foundational physical principles, we will consider how these justificatory strategies evolved in the nineteenth century. This means focusing on two developments: the onset of non-Euclidean geometry and its relation to kinematics, and the ongoing attempts to justify force-laws on a priori grounds, culminating in Heinrich Hertz's geometrization of force. We will begin with Helmholtz's reinterpretation of geometry as a kinematic science of rigid-body motion, which he termed 'physical geometry'.

Hilary Putnam once suggested that the overthrow of Euclidean geometry was 'the most important event in the history of science for the epistemologist' (Putnam 1979, p. x). If the epistemologist is a Kantian, then Hermann von Helmholtz is his beast. Active in most domains of nineteenth-century science, Helmholtz gradually developed a naturalistic epistemology in stark contrast to the Kantian approach of his youth, and this epistemology, when joined to his work in physiology, mathematics, physics, and medicine, was instrumental in establishing the naturalistic world-view taken for granted by most scientists today. Nonetheless, I am not going to approach Helmholtz's work from that point of view in what follows, because I am primarily interested in the fate of our a priori principles. Both as a young Kantian and an older empiricist, Helmholtz continued to work within this tradition of rationalist mechanics, combining it only partly with his empiricist epistemology.

6.2.1 Geometry and Kinematics

In a series of papers from 1868–78, Helmholtz attacked Kant's theories of space and time by arguing that (1) the purpose of geometry was to measure spatial forms, and that this could be achieved only under the condition that rigid bodies (rulers) were freely transportable in space, (2) since the transportation of rigid bodies is a material process, the underlying assumptions of geometry are empirical and kinematic. Furthermore, (3) the possibility of rigid-body displacements conforming to non-Euclidean laws meant that the question of which geometry to use was an empirical one. In particular, while a "strict Kantian" might preserve the a priori validity of Euclidean geometry by *defining* rigidity in Euclidean terms, this would render it analytic, and not synthetic, as Kant had clamed. Finally, we cannot know in advance whether the equal parts of space thus defined would