Department of Mathematics and Statistics
University of Ottawa

MAT 2377C

FINAL EXAM PRACTICE

10 December 2015
Time: 180 minutes

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Student Number: ________________

Family Name: ________________

First Name: ________________

This is a closed book examination. Only non-programmable and non-graphic calculators are permitted. **Record your answer to each question in the table below.** Your package includes the title page, six pages with questions, the formula sheet, normal and t-tables. Number of questions: **24**. **NOTE: At the end of the examination, hand in only this page.**

Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Your signature: ________________________________

Question	Answer	Question	Answer
1		13	
2		14	
3		15	
4		16	
5		17	
6		18	
7		19	
8		20	
9		21	
10		22	
11		23	
12		24	

GOOD LUCK !!!

Q1. Suppose that for a very large shipment of integrated-circuit chips, the probability of failure for any one chip is 0.09. Find the probability that at most 2 chips fail in a random sample of size 20. (The numbers are rounded down to the second decimal place).

(a) 0.13 (b) 0.90 (c) 0.73 (d) 0.20 (e) none of the preceding

Solution to Q1:

$X \sim B(20, 0.09)$. So

$$P(X \leq 2) = \binom{20}{0}(.09)^0(.91)^{20} + \binom{20}{1}(.09)^1(.91)^{19} + \binom{20}{2}(.09)^2(.91)^{18} = 0.73$$

Q2. Assume that in a box containing 100 items, five of them are defective. We select 20 items without replacement. What is the probability that at most one of them will be defective?

(a) 0.319 (b) 0.420 (c) 0.453
(d) 0.739 (e) none of the preceding

Solution to Q2:

Let X be the number of defective items. We have

$$P(X = 0) + P(X = 1) = \frac{\binom{5}{0}\binom{95}{20}}{\binom{100}{20}} + \frac{\binom{5}{1}\binom{95}{19}}{\binom{100}{20}} = 0.739.$$

Q3. A company which produces a particular drug has two factories, A and B. 30% of the drug are made in factory A, 70% in factory B. Suppose that 95% of the drugs produced by the factory A meet specifications while only 75% of the drugs produced by the factory B meet specifications. If I buy the drug, what is the probability that it meets specifications?

(a) 0.81 (b) 0.95 (c) 0.75
(d) 0.7 (e) none of the preceding

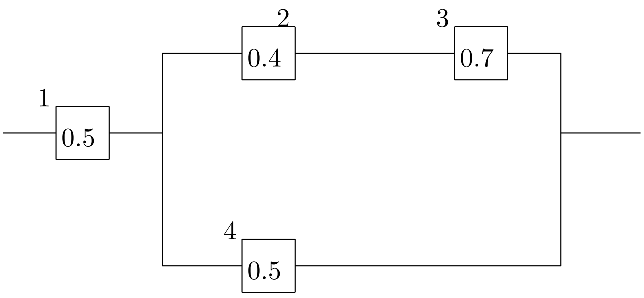
Solution to Q3:

M - meets specifications, A - produced by A, B - produced by B. Given: $P(A) = 0.3$, $P(B) = 0.7$, $P(M|A) = 0.95$, $P(M|B) = 0.75$. To find: $P(M)$. Use the total probability rule:

$$P(M) = P(M|A)P(A) + P(M|B)P(B) = 0.81$$

The answer a) was corrected

Q4. Consider the following system with four components. We say that it is functional if there exists a path of functional components from left to right. The probability of each component functions is shown. Assume that the components function or fail independently. What is the probability that the system does not operate ?



(a) 0.32 (b) 0.16 (c) 0.035 (d) 0.68 (e) none of the preceding

Solution to Q4:

Call 'Box B' - components 2,3,4, 'Box C' - components 2,3.

$$\begin{aligned} P(\text{Box C operates}) &= P(\text{component 2 operates and component 3 operates}) \\ &= P(\text{component 2 operates})P(\text{component 3 operates}) = 0.4 \times 0.7 = 0.28. \end{aligned}$$

$$\begin{aligned} P(\text{Box B operates}) &= P(\text{Box C operates or component 4 operates}) \\ &= P(\text{Box C operates}) + P(\text{component 4 operates}) - \\ &\quad P(\text{Box C operates})P(\text{component 4 operates}) \\ &= 0.28 + 0.5 - 0.28 * 0.5 = 0.64. \end{aligned}$$

$$\begin{aligned} P(\text{system operates}) &= P(\text{component 1 and Box B operate}) \\ &= P(\text{component 1 operates})P(\text{Box B operates}) \\ &= 0.5 * 0.64 = 0.32. \end{aligned}$$

Thus, $P(\text{system does not operate}) = 0.68$

Q5. Assume that X is a discrete random variable with the following probability function:

$$f_X(x) = \begin{cases} k & \text{if } x = 0 \text{ or } 1 \\ \frac{14}{25} & \text{if } x = 3 \\ 0 & \text{otherwise.} \end{cases}$$

The value of k is

(a) 11/50 (b) 11/25 (c) 22/25

(d) 22/50 (e) None of the preceding.

Solution to Q5:

$P(X = 0) + P(X = 1) + P(X = 3) = 1$, so that $k + k + 14/25 = 1$.

Q6. It is known that a particular company produces 30% defective items. 10 items are selected randomly. Calculate probability that at most two of them are defective.

(a) 0.3828 (b) 0.0014 (c) 0.2451
(d) 0.7549 (e) None of the preceding.

Solution to Q6:

Let $X \sim Bin(10, .3)$. Compute $P(X \leq 2)$.

$$P(X = 0) + O(X = 1) + P(X = 2)$$

Use the binomial formula.

Q7. Assume that times of failures of a computer network can be modeled using a Poisson process with mean 10 failures per year. What is the standard deviation of the waiting time to the first failure? (Time is measured as the part of a year, for example 0.500 means half a year).

(a) 0.222 (b) 0.1 (c) 0.333
(d) 0.471 (e) none of the preceding

Solution to Q7:

Waiting time to the first failure, X, is exponential $\lambda = 10$. Standard deviation is $\sqrt{\text{Var}(X)} = \sqrt{1/\lambda^2} = \sqrt{1}/10 = 0.1$.

Q8. Assume that X is a continuous random variable with the density f_X:

$$f_X(x) = \begin{cases} Ke^{-3x} & \text{if } x \geq -\ln(3)/3, \\ 0 & \text{otherwise.} \end{cases}$$

The value of K is:

(a) $\frac{\ln 3}{3}$ (b) $-\frac{\ln 3}{3}$ (c) 1
(d) 10 (e) None of the preceding.

Solution to Q8:

$$1 = \int_a^\infty f_X(x)\, dx = \int_{-\ln(3)/3}^\infty K \exp(-3x)\, dx = -\frac{1}{3} \exp(-3x)|_{x=-\ln(3)/3}^\infty \times K = K \text{ so that } K = 1.$$

Q9. The air pressure in a randomly selected tire put on a certain model new car is normally distributed with mean value 31 lb/in^2 and standard deviation 0.5 lb/in^2. What is the probability that the pressure for a randomly selected tire is between 30.5 and 31.5 lb/in^2.

(a) 0.6827 (b) 0.3173 (c) 0.5000
(d) 0.4245 (e) none of the preceding

Solution to Q9:

Let X be air pressure of a randomly selected tire. We have $X \sim N(31, 0.5^2)$. Therefore,

$$P(30.5 < X < 31.5) = P\left(\frac{30.5 - 31}{0.5} < Z < \frac{31.5 - 31}{0.5}\right) = P(-1 < Z < 1) = 0.6827$$

Q10. Assume that we have a sample $X_1, \ldots, X_{100}$ from an arbitrary distribution with mean $\mu = 1$ and variance $\sigma^2 = 4$. The approximate probability that the sample mean exceeds 1.5, is:

(a) 0.99379 (b) 0.00621 (c) 0.5000 (d) 0.7742 (e) none of the preceding

Solution to Q10:

$$P(\bar{X} > 1.5) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \frac{1.5 - 1}{2/10}\right) = P(Z > 2.5) = 1 - 0.993790 = 0.00621$$

Q11. Assume that we have a sample of size 10 from a population $N(4, 9)$. Denote by $\overline{X}$ and S^2, the sample mean and sample variance, respectively. Find c such that

$$P\left(\frac{\overline{X} - 4}{S/\sqrt{10}} \geq c\right) = 0.01.$$

(a) 1.833 (b) 2.821 (c) 1.645
(d) 2.424 (e) none of the preceding

Solution to Q11:

Equivalent statement: find c such that

$$P\left(\frac{\overline{X} - 4}{S/\sqrt{10}} \geq c\right) = 0.01.$$

We have that $\frac{\overline{X} - 4}{S/\sqrt{10}}$ has Student distribution with $n - 1 = 9$ degrees of freedom. From the table we read that $P(t_9 > 2.821) = 0.01$, thus $c = 2.821$.

Q12. Assume we have a sample of size $n = 5$: $5, 34, 12, 10, 4$. The sample mean, the sample variance and the sample median are, respectively:

(a) 13, 149 and 10 (b) 13, 119.2 and 12 (c) 12, 119.2 and 5
(d) 13, 12.2 and 4.5 (e) none of the preceding

Solution to Q12:

$\overline{x} = \sum x_i/n = 65/5 = 13$ and

$$s^2 = \frac{(\sum x_i^2) - (\sum x_i)^2/n}{n - 1} = \frac{1141 - (65)^2/5}{4} = 149$$

Q13. The engineer wants to construct the (two-sided) confidence interval for the mean such that the precision $E = 0.2$. If $\alpha = 0.1$ and $\sigma = 4$, what should be the sample size?

(a) 25 (b) 1537 (c) 423

(d) 1083 (e) None of the preceding.

Solution to Q13:

$$n = \left[\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2\right] + 1 = \left[\left(\frac{4 * 1.645}{0.2}\right)^2\right] + 1 = 1083$$

In the original version I wrote incorrectly one-sided.

Q14. In a random sample of 100 homes in Halifax, 23 are found to be heated by electricity. Construct a 90% confidence interval on the true proportion of homes heated by electricity.

(a) [0.1608, 0.2992] (b) [0.1508, 0.2992]

(c) [0,1] (d) [0.1608,0.3145]

(e) none of the preceding

Solution to Q14:

$$\hat{p} \pm z_{0.1/2}\sqrt{\hat{p}(1-\hat{p})/n} = 0.23 \pm 1.645\sqrt{0.23 * 0.77/100} = [0.1608, 0.2992]$$

Q15. The engineer measures $n = 25$ pieces of steel and obtains $\bar{x} = 6$. The weight follows normal distribution with known variance $\sigma^2 = 16$. The two-sided 99% confidence interval for the mean is:

(a) (-0.272,12.272) (b) (3.250,8.750)

(c) (4.432,7.568) (d) (3.940,8.060)

(e) None of the preceding.

Solution to Q15:

$$(\bar{x} - z_{\alpha/2}\sigma/\sqrt{n}, \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}) = (6 - 2.575 * 4/5, 6 + 2.575 * 4/5) = (3.94, 8.06)$$

Q16. A machine is producing metal pieces that are cylindrical in shape. A sample of pieces is taken and the diameters are

```
1.01 0.97 1.03 1.04 0.99 0.98  0.99 1.04 1.03 1.01
```

Find the two-sided 99% confidence interval for the true mean diameter. Assume that the population is normally distributed.

(a) [0.989,1.022] (b) [0.983,1.035] (c) [0.991,1.034]

(d) [.987,1.024] (e) none of the preceding.

Solution to Q16:

The sample mean and standard deviation are $\overline{x} = 1.009$ and $s = 0.0256$, respectively.
We have $t_{.005,10-1} = 3.250$. So

$$\overline{x} \pm 3.250 \frac{s}{\sqrt{n}} = [0.983, 1.035].$$

Q17. The engineer measures $n = 25$ pieces of steel and obtains $\bar{x} = 6$. The weight follows normal distribution with known variance $\sigma^2 = 16$. He wants to test $H_0 : \mu = 5$ against $H_1 : \mu > 5$. The p-value for the test is:

(a) 0.0500 (b) 0.1057 (c) 0.8943

(d) 1.000 (e) None of the preceding.

Solution to Q17:

$$P(\bar{X} > 6) = P\left(Z > \frac{6-5}{4/5}\right) = P(Z > 1.25) = 1 - 0.8943 = 0.1057.$$

Q18. A company claims that the mean deflection of a piece of steel which is 10 feet long, is equal to 0.012. A buyer suspects that it is bigger than 0.012. The following data has been collected:

0.0132 0.0138 0.0108 0.0126 0.0136 0.0112 0.0124 0.0116 0.0127 0.0131

Assume normality. *Hint:* $\sum_{i=1}^{n} x_i^2 = 0.0016$. The p-value for the appropriate one-sided test and the decision are:

(a) $p \in (0.05, 0.1)$. Reject H_0 for $\alpha = 0.05$. (b) $p \in (0.05, 0.1)$. Do not reject H_0 for $\alpha = 0.05$.

(c) $p \in (0.1, 0.25)$. Reject H_0 for $\alpha = 0.05$. (d) $p \in (0.1, 0.25)$. Do not reject H_0 for $\alpha = 0.05$.

(e) None of the preceding.

Solution to Q18:

The estimated variance is

$$S^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2\right) = 1.02 * 10^{-6}.$$

The observed mean is $\bar{x} = 0.0125$. We calculate the p-value:

$$P\left(\bar{X} > 0.0125\right) = P\left(\frac{\bar{X}-\mu}{S/\sqrt{n}} > \frac{0.0125 - 0.012}{\sqrt{1.02 * 10^{-6}/10}}\right)$$
$$= P(t_9 > 1.5638) \in (0.05, 0.1).$$

Do not reject H_0 for $\alpha = 0.05$.

Q19. A company is currently using titanium alloy rods it purchases from supplier A. A new supplier (supplier B) approaches the company and offers the same quality (at least according to supplier Bs claim) rods at a lower price. The company is certainly interested in the offer. At the same time, the company wants to make sure that the safety of their product is not compromised. The company randomly selects ten rods from each of the lots shipped by suppliers A and B and measures the yield strengths of the selected rods. The observed sample mean and sample standard deviation are 651 MPa and 2 MPa for suppliers A rods, respectively, and the same parameters are 657 MPa and 3 MPa for supplier Bs rods. The company tests the hypotheses $H_0 : \mu_A = \mu_B$ against $\mu_A \neq \mu_B$. For $\alpha = 0.05$ the decision is:

(a) Reject H_0;
(b) Do not reject H_0;
(c) none of the preceding

Assume equal variances.

Solution to Q19:

I added "assume equal variances" statement so that you can use case 2. I also modified the statement, to be in line with what I taught.

See Assignment 6

Q20. In an effort to compare the durability of two different types of sandpaper, 10 pieces of type A sandpaper were subjected to treatment by a machine which measures abrasive wear. Eleven pieces of type B sandpaper were subjected to the same treatment.

x_{1i}	27	26	24	29	30	26	27	23	28	27	
x_{2i}	24	23	22	27	24	21	24	25	24	23	20

Note: $\sum_i x_{1i} = 267$, $\sum_i x_{2i} = 257$, $\sum_i x_{1i}^2 = 7169$, $\sum_i x_{2i}^2 = 6041$. From this you can easily compute sample means and sample variances, for example

$$s_1^2 = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_{i1} - \bar{x}_1) = \frac{\sum_{i=1}^{n_1} x_{i1}^2 - \frac{1}{n_1}\left(\sum_{i=1}^{n_1} x_{i1}\right)^2}{n_1 - 1}.$$

Assuming normality and equality of variance, we want to test for equality of mean abrasive wear. The appropriate p-value is:

(a) $p < 0.002$
(b) $p > 0.2$
(c) $p \in (0.05, 0.1)$
(d) $p \in (0.1, 0.2)$
(e) None of the preceding

Solution to Q20:

This is two sample test. $H_0 : \mu_1 = \mu_2$, $H_1 : \mu_1 \neq \mu_2$. We compute $S_1^2 = 4.45$, $S_2^2 = 3.65$, $S_p^2 = 4.03$, $\bar{X}_1 = 26.71$ $\bar{X}_2 = 23.36$. The p-value is

$$2P\left(\bar{X}_1 - \bar{X}_2 > 3.34\right) = 2P\left(\frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{1/n_1 + 1/n_2}} > \frac{3.34}{\sqrt{4.03}\sqrt{1/10 + 1/11}}\right) = 2P(t_{19} > 3.8037) < 0.002$$

Q21. Ten engineers' knowledge of basic statistical concepts was measured on a scale of 100 before and after a short course in statistical quality control. The result are as follows:

Engineer	1	2	3	4	5	6	7	8	9	10
Before X_{1i}	43	82	77	39	51	66	55	61	79	43
After X_{2i}	51	84	74	48	53	61	59	75	82	53

Let μ_1 and μ_2 be the mean mean score before and after the course. We want to test $H_0 : \mu_1 = \mu_2$ against $\mu_1 < \mu_2$. The is (use $\alpha = 0.05$)

(a) Do not reject H_0;

(b) Reject H_0;

(c) none of the preceding

Solution to Q21:

The question has been modified to be in line with what I taught

The differences $D_i = X_{1i} - X_{2i}$ are:

Engineer	1	2	3	4	5	6	7	8	9	10
Before X_{1i}	43	82	77	39	51	66	55	61	79	43
After X_{2i}	51	84	74	48	53	61	59	75	82	53
Difference D_i	-8	-2	3	-9	-2	5	-4	-14	-3	-10

so that $\bar{D} = -4.4$, $S_D = 5.91$. We calculate the p-value:

$$P\left(\bar{D} < -4.4\right) = P\left(\frac{\bar{D}}{S_D/\sqrt{n}} < \frac{-4.4}{5.91/\sqrt{(10)}}\right) = P(t_9 < -2.35) = P(t_9 > 2.35) \in (0.02, 0.025)$$

Reject for $\alpha = 0.05$

Q22. The following output was produced with `t.test` command in `R`

```
One Sample t-test

data:  x
t = 32.9198, df = 999, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.9644462 1.0867155
sample estimates:
mean of x
 1.025581
```

Based on this output, which statement is correct:

(a) If the type I error is 0.05, then we reject $H_0 : \mu = 0$ in favour of $H_1 : \mu > 0$;

(b) If the type I error is 0.05, then we reject $H_0 : \mu = 0$ in favour of $H_1 : \mu \neq 0$;

(c) If the type I error is 0.01, then we reject $H_0 : \mu = 0$ in favour of $H_1 : \mu > 0$;

(d) If the type I error is 0.01, then we reject $H_0 : \mu = 0$ in favour of $H_1 : \mu < 0$;

(e) None of the preceding

Q23. Regression methods were used to analyze the data from a study investigating the relationship between roadway surface temperature in F (x) and pavement defection (y). Summary quantities were $n = 20$, $\sum y_i = 12.575$, $\sum y_i^2 = 8.86$, $\sum x_i = 1478$, $\sum x_i^2 = 143,215.8$, $\sum x_i y_i = 1083.67$. The slope coefficient is:

(a) 0.004 (b) 0.68 (c) 0.60 (d) 0.4 (e) none of the preceding

Solution to Q23:

We have $S_{xy} = 141.445$, $S_{xx} = 33991.6$, $S_{yy} = 0.731875$;

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x},$$

where

$$\boxed{\begin{aligned} S_{xy} &= \sum_{i=1}^{n} x_i y_i - \frac{1}{n}(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i) \\ S_{xx} &= \sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2 \\ S_{yy} &= \sum_{i=1}^{n} y_i^2 - \frac{1}{n}(\sum_{i=1}^{n} y_i)^2 \end{aligned}}$$

So $\hat{\beta}_1 = 0.00416$

Q24. Consider the data from Question 23. The correlation coefficient is:

(a) 0.9; there is no linear relationship between x and y;

(b) 0.9; there is a linear relationship between x and y;

(c) 0.1; there is no linear relationship between x and y;

(d) 0.1; there is a linear relationship between x and y;

(e) none of the preceding

Solution to Q24:

This is the last question

Solutions to multiple choice questions:

Q1 $\longrightarrow$ c
Q2 $\longrightarrow$ d
Q3 $\longrightarrow$ a
Q4 $\longrightarrow$ d
Q5 $\longrightarrow$ a
Q6 $\longrightarrow$ a
Q7 $\longrightarrow$ b
Q8 $\longrightarrow$ c
Q9 $\longrightarrow$ a
Q10 $\longrightarrow$ b
Q11 $\longrightarrow$ b
Q12 $\longrightarrow$ a
Q13 $\longrightarrow$ d
Q14 $\longrightarrow$ a
Q15 $\longrightarrow$ d
Q16 $\longrightarrow$ b
Q17 $\longrightarrow$ b
Q18 $\longrightarrow$ b
Q19 $\longrightarrow$ a
Q20 $\longrightarrow$ a
Q21 $\longrightarrow$ b
Q22 $\longrightarrow$ b
Q23 $\longrightarrow$ a
Q24 $\longrightarrow$ b