Common agency with risk-averse agent

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In a common agency model with a risk-averse agent and private information distortion in the equilibrium policy from the first-best is greater compared to the case of a risk-neutral agent. The principals are unable to screen completely the agent’s preferences if he is sufficiently risk-averse: there is bunching in the contract. The contribution schedules keep track of informational externality. However, when the coefficient of risk-aversion goes to zero the contributions become truthful as in the complete information case.

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1. Introduction

We consider a common agency model where the risk-neutral principals contract with a risk-averse agent. Such models arise naturally in situations where the actions of one party affect the well-being of a few players (examples of such models in industrial economics are given in Laffont and Tirole, 1993, applications to political economy in Grossman and Helpman, 2001). In these models, the agent is characterized by privately observed type which is one-dimensional and continuously distributed. Contracting takes place ex-ante when both parties have symmetric but incomplete information.

The modeling of common agency under complete information was first developed in the seminal paper of Bernheim and Whinston (1986) where the notion of truthful equilibria was introduced. In these equilibria, the contributions reflect the marginal valuations of the principals everywhere. Such contributions implement the first-best and essentially are the only coalition-proof equilibria. Subsequently, the models of common agency under incomplete information were developed. They focused on two polar cases of the agent’s risk-aversion: zero risk-aversion (ex-ante contracting with risk-neutral agent) and infinite risk-aversion (ex-post contracting with the risk-neutral agent). For zero risk-aversion Laussel and Le Breton (1998) show that contracting results in an efficient output that is implemented by truthful contributions. Conversely, for ex-post contracting the output is not efficient, however it is strictly monotonic. As Dixit (1996) and Martimort (1996) point out, there are transaction costs in the models of political influence which preclude the efficient output. Moreover, Martimort and Stole (2009a) show that the contributions always keep track of asymmetry of information and even in the limit (when the uncertainty about the agent’s type disappears) they are not truthful. Even though two polar cases of zero and infinite risk-aversion are realistic in certain environments, there is a need to consider the general case of intermediate risk-aversion.
In this paper we offer a unified approach to modeling common agency by considering two risk-neutral principals who contract with a risk-averse agent. To capture different attitudes towards risk, the agent is assumed to have a CARA utility with the parameter of absolute risk-aversion $\sigma$. For example, previously studied models of common agency under asymmetric information can be described as ex-ante contracting when $\sigma = 0$ and ex-post contracting when $\sigma = \infty$.

We show that the distortion in output from the first-best is aggravated due to a free-riding problem in the common agency compared to the one-principal (merged) case. This distorted output for sufficiently large values of risk-aversion is no longer strictly monotonic: bunching occurs for less efficient types. Interestingly, the contributions that support this output keep track of the principals’ marginal valuations, and in the limiting case when risk aversion is zero, they are truthful.

In principal agent literature risk-aversion of the agent did not receive much attention. All research in this area has focused on the one-principal models. In the case of two principals considered in this paper, the bunching result is similar to the result in the context of public regulation with one-principal. Baron and Besanko (1987) show that randomness in cost or noisy monitoring of supplier’s deterministic cost are no longer equivalent when one considers a risk-averse supplier, contrary to the case of a risk-neutral agent.

The paper is organized as follows. In Section 2, we introduce the common agency model with the risk-averse agent. Section 3 shows the equilibrium in the one-principal model with adverse selection and the risk-averse agent. Section 4 presents polar cases of risk neutrality and infinite risk-aversion and shows different techniques which are needed to solve these cases. Our main results are presented in Section 5. Most of proofs are relegated to Appendix A.

2. The model

Two principals $P_i$, $i = 1, 2$, contract with a common agent $A$ on a one-dimensional output $q$ and monetary contributions $t_i$. The principals $P_i$ are risk-neutral with utility $S_i(q) - t_i$, where the surplus is $S_i(q) = v_i q - q^2/4$ and $t_i$ is the monetary contribution made to the common agent. The valuations $v_i$, $i = 1, 2$, are common knowledge and $v = \sum_{i=1}^{2} v_i$ denotes a joint valuation of the good by the principals. The agent (he) produces the public good $q$ at cost $C(q, \theta) = \theta q$, where $\theta$ is the cost parameter. Contracting takes place ex-ante, when both principals and the agent have symmetric information about $\theta$. The parameter $\theta$, from the players’ ex-ante perspective, is uniformly distributed on the interval $[\theta, \bar{\theta}]$.

The agent’s ex-post payoff is $s = \sum_{i=1}^{2} t_i - \theta q$, and his utility has CARA functional form:

$$V(s) = 1 - e^{-\sigma s},$$

where the coefficient of absolute risk-aversion $\sigma \in \mathbb{R}_+ \cup \{\infty\}$ characterizes different attitude towards risk. For example, if $\sigma = 0$, the agent is risk-neutral and if $\sigma = \infty$, he is infinitely risk-averse.

The timing of the model is the following:

• The principals simultaneously offer the contribution schedules to the agent, i.e., the principal $P_i$ offers $t_i(\cdot)$.
• The agent after observing the principals’ offers chooses to accept some offers, or reject all.\(^1\) In case of rejection the principals’ reservation utility of the agent and principals is zero.
• The parameter $\theta$ is realized by the agent.
• The agent produces $q \geq 0$, and the principals pay contributions $t_1(q)$ and $t_2(q)$.

We denote by $U_i(\theta)$ the information rent of the agent with the cost parameter $\theta$ when accepting both contracts:

$$U(\theta) = \max_{q \geq 0} \left\{ \sum_{i=1}^{2} t_i(q) - \theta q \right\},$$

and we define the output in this case as $q(\theta)$. Let $U_i(\theta)$ be the agent’s information rent and $q_i(\theta)$ the optimal output when contracting only with the principal $P_i$:

$$U_i(\theta) = \max_{q \geq 0} t_i(q) - \theta q \quad \text{and} \quad q_i(\theta) = \arg\max_{q \geq 0} t_i(q) - \theta q.$$  

(1)

The equilibrium concept used in the game is subgame-perfect Nash equilibrium and we focus on pure strategy equilibria. Thus, the equilibrium consists of 3-tuple $(t_1(q), t_2(q), q(\theta))$, such that the output schedule $q(\theta)$ chosen by the agent maximizes his payoff:

$$q(\theta) = \arg\max_{q \geq 0} \left\{ \sum_{i=1}^{2} t_i(q) - \theta q \right\}, \quad \text{for all} \quad \theta \in [\theta, \bar{\theta}],$$

(2)

\(^1\) This is the case of the delegated common agency. In the case of the intrinsic common agency the agent has only two options: accept all offers or none.
and the strategy \( t_i(q) \) of the principal \( P_i \) is the best-response to the contribution schedule \( t_{-i}(q) \) offered by the principal \( P_{-i} \), given the agent’s choice in the last stage. We consider competition between the principals via piece-wise differentiable contribution schedules \( t_i(q), i = 1, 2 \) (this is without loss of generality, see Peters, 2001; Martimort and Stole, 2002). The agent after observing the principals’ offers chooses a piece-wise differentiable output \( q(\theta) \). This specification of players’ strategy spaces allows us to write the conditions for equilibrium of the game. In the last stage of the game, the choice of the agent at state \( \theta \), assuming that he accepts both contracts is determined by the first-order condition\(^2\)

\[
\sum_{i=1}^{2} t'_i(q(\theta)) = \theta, \quad \text{for all } \theta \in [\theta, \bar{\theta}],
\]

where \( t'_i(q(\theta)) \) denotes the derivative of \( t_i \) with respect to \( q \).\(^3\) Given the contribution schedule \( t_{-i}(q) \), the principal’s \( P_i \) best-response problem can be formulated as

\[
\max_{q(\theta), U(\theta)} \int_{\theta}^{\bar{\theta}} [S_i(q(\theta)) - U(\theta) + t_{-i}(q(\theta)) - \theta q(\theta)] \frac{d\theta}{\Delta \theta},
\]

subject to the first-order condition (3), implementability conditions

\[
\dot{U}(\theta) = -q(\theta), \quad \dot{q}(\theta) \leq 0,
\]

the ex-ante participation constraint for the agent

\[
E_\theta V(U(\theta)) = \int_{\theta}^{\bar{\theta}} V(U(\theta)) \frac{d\theta}{\Delta \theta} \geq 0,
\]

and the ex-ante full participation constraint

\[
E_\theta V(U(\theta)) = \int_{\theta}^{\bar{\theta}} V(U(\theta)) \frac{d\theta}{\Delta \theta} \geq E_\theta V(U_{-i}(\theta)) = \int_{\theta}^{\bar{\theta}} V(t_{-i}(q_{-i}(\theta)) - \theta q_{-i}(\theta)) \frac{d\theta}{\Delta \theta}.
\]

This ex-ante full participation constraint ensures that the agent prefers to accept both contracts rather than only the one offered by \( P_{-i} \).

As a benchmark, we note that the efficient production takes place under complete information when all parties (both principals and the agent) form a grand coalition. In this case, the first-best output at state \( \theta \) maximizes a joint surplus minus the cost of production as given by the following Lindahl-Samuelson condition:

\[
\sum_{i=1}^{2} S_i(q^{FB}(\theta)) = \partial_1 C(q^{FB}(\theta), \theta),
\]

which leads to \( q^{FB}(\theta) = \nu - \theta \). The joint contribution schedule which supports this first-best output is \( t(q) = (\nu - q)q \). To insure first-best provision for any value of \( \theta \), we assume that \( \nu = v_1 + v_2 > \bar{\theta} \).

3. One-principal case

Let us start with the model where two principals perfectly coordinate their strategies. This would be the case when both principals merge (or collude). In this case, we consider one-principal who represents the merger. This principal has aggregated utility \( \nu q - q^2/2 - t(q) \) and offers the contribution schedule \( t(q) \) to the agent who decides whether or not to accept this contract ex-ante.\(^4\) The principal’s problem becomes

\[
\max_{q(\theta), U(\theta)} \int_{\theta}^{\bar{\theta}} [vq(\theta) - \frac{q^2(\theta)}{2} - \theta q(\theta) - U(\theta)] \frac{d\theta}{\Delta \theta},
\]

subject to (5) and (6).

The following characterization of the equilibrium is due to Salanié (1990).

**Proposition 1** (Salanié 1990). There exists \( \delta_{FB} \geq 0 \) such that the equilibrium in the one-principal case has the form:

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\(^2\) We assume a strict concavity of agent’s objective function. The concavity is verified ex-post.

\(^3\) The derivatives with respect to \( \theta \) we denote by dot.

\(^4\) This problem was considered in Salanié (1990) where the contribution schedule \( t(q) \) is chosen as a state variable when solving the induced optimal control problem. Instead it is convenient for our purposes to use the agent’s monetary payoff \( U(\theta) \) as a state variable since this approach can be easily generalized to the case of two principals where there are two contribution schedules but still one monetary payoff of the agent.
(1) If \( \sigma \leq \hat{\sigma}_M \), then the equilibrium output \( q(\theta) \) is the unique solution of the differential equation
\[
\dot{q}(\theta) = \sigma q(\theta)(\dot{q}(\theta) + 2),
\]
\[q(\theta) = q^{FB}(\theta), \quad 0 > q(\theta) > -2.
\]

This output \( q(\theta) \) is implemented via contributions \( t(\theta) = \theta q(\theta) - \frac{1}{\theta} \log(q(\theta) + 2) \);

(2) If \( \sigma > \hat{\sigma}_M \), then in the equilibrium, there exist constants \( \hat{\theta}_M(\sigma) \) and \( \bar{\theta}_M(\sigma) \) such that
(i) for \( \theta \in [\hat{\theta}_M(\sigma), \bar{\theta}_M(\sigma)] \), the output \( q(\theta) = \hat{q}_M(\sigma) \);
(ii) for \( \theta \in [\hat{\theta}_M(\sigma), \bar{\theta}_M(\sigma)] \), the output \( q(\theta) \) is the unique solution of the differential equation
\[
\dot{q}(\theta) = \sigma q(\theta)(\dot{q}(\theta) + 2),
\]
\[q(\theta) = q^{FB}(\theta), \quad 0 > q(\theta) > -2.
\]

This proposition shows that separation in outputs prevails for the most efficient values of the cost parameter \( \theta \). For sufficiently large values of the absolute risk-aversion the principal may not separate the output for less efficient values: the contract exhibits some bunching.

4. Common agency model under zero and infinite risk-aversion

In this section, we consider the common agency modeling under the two polar scenarios of the agent’s risk-aversion: agent is risk-neutral (zero risk-aversion) and infinite risk-aversion.

4.1. Risk neutrality, \( \sigma = 0 \)

Conceptually, the common agency model under ex-ante contracting with the risk-neutral agent is similar to the case of complete information (Laussel and Le Breton, 1998). In this complete information framework, Bernheim and Whinston (1986) introduced truthful contribution schedules.

Definition 1 (Bernheim and Whinston 1986). The contribution schedule
\[
t_i(q) = S_i(q) - C_i = v_i q - \frac{q^2}{4} - C_i,
\]
where \( C_i \) is a constant is called truthful.

The equilibrium \((t_1(q), t_2(q), q(\theta))\), where \( t_i(q), i = 1, 2 \) are truthful contribution schedules, is called truthful equilibrium. To motivate this choice of equilibrium concept, notice that these equilibria are the only coalition-proof equilibria. Moreover, these contributions at margin coincide everywhere with the principals’ valuations and, therefore, the agent’s preferences, after accepting both contracts, represent aggregate preferences for a grand coalition. Thus, the output is efficient.

Under ex-ante contracting the properties of the equilibrium payoffs can be described in terms of the cooperative game \((S, W_S)\) with the characteristic function \( W_S = E_\theta W_S(\theta) \) (see Laussel and Le Breton, 1998; Martimort and Semenov, 2007), where \( W_S(\theta) \) represents the joint surplus of the principals from the set \( S \in \{1, 2, (1, 2)\} \) and the agent with the cost parameter \( \theta \): \( W_S(\theta) = \max_{q \geq 0} \sum_i S_i(q - \theta) \). The next proposition characterizes the set of truthful equilibria.

Proposition 2. In any truthful equilibrium of the common agency game under risk neutrality, \( \sigma = 0 \), the agent’s optimal choice is the first-best efficient output and the equilibrium contributions are non-negative in expected value:

1. **Non-congruent principals**: If the valuations of the principals satisfy \( v < v^* \), the cooperative game \((S, W_S)\) is sub-additive and the equilibrium is unique. The equilibrium contributions (9) are truthful with the constants \( C_i \) determined by
\[
C_i = \int_{\hat{\theta}}^{\bar{\theta}} \left(-\frac{(v - \theta)^2}{2} + (v_{i-1} - \theta)^2\right) d\theta / \Delta \theta.
\]
In this equilibrium the agent obtains a positive rent: \( \int_{\hat{\theta}}^{\bar{\theta}} W_{12}(\theta) d\theta / \Delta \theta = \sum_{i=1}^{v^*} W_i - W_{12} > 0. \) The payoff of the principal \( P_i \) is \( C_i \).

2. **Congruent principals**: If \( v \geq v^* \), the cooperative game \((S, W_S)\) is superadditive. There is a continuum of truthful equilibria and the agent obtains rent in any of them.

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\( v^* = \frac{1}{2 \sigma (v_1 - v_2)^2 + \sigma^2 + \bar{\theta}^2 + \hat{\theta}^2). \)
Proof. See Appendix A. □

To see the intuition behind this proposition, note that the conflict of interests, when the principals have very different valuations (i.e., $v_1 - v_2$ is large), does not allow them to extract all the rent. However, the less dispersed the interests of the principals, the more successful they are in extracting the agent’s surplus. In this case, they act as one entity which extracts all the rent, and then they redistribute the joint surplus. Since there are many ways to redistribute the total surplus, there is a multiplicity of equilibria in the game.

4.2. Infinite risk-aversion, $\sigma = \infty$

Consider now the common agency model with infinitely risk-averse agent. In contrast to the case when $\sigma = 0$, contracting with infinitely risk-averse agent does not allow for negative payoffs for the agent in any state of the world. This leads to the ex-post participation constraint $U(\theta) \geq 0$, for all $\theta$. Therefore, contributions in this case must be non-negative ($t_i \geq 0$) in and out-of-equilibrium. Also, the ex-post full participation constraint replaces (7). Since contributions are non-negative, we have $U_{-i}(\theta) \geq 0$ for all $\theta$ and the constraint $U(\theta) \geq 0$ follows from $U(\theta) \geq U_{-i}(\theta)$. Given the contribution schedule $t_{-i}(q)$, this constraint is binding at $\hat{\theta}$ and we obtain from the incentive compatibility constraint the following:

$$U(\theta) = \int_{\theta}^{\hat{\theta}} q(\theta) d\theta + U_{-i}(\hat{\theta}),$$

(10)

where $U_{-i}(\hat{\theta})$ is determined by (1) and, therefore, does not depend on the strategy of $P_i$. Inserting the expression of $U(\theta)$ into the objective function for the program (4) and differentiating point-wise, we obtain

$$v_i - q(\theta) + t'_{-i}(q(\theta)) - 2\theta + \theta = 0.$$

Using the first-order condition (3), the marginal contribution in the equilibrium range are defined by

$$t'_{i}(q(\theta)) = v_i - q(\theta) - (\theta - \theta).$$

(11)

Summing these marginal contributions and using the agent’s first-order condition (3), we have the equilibrium output given by

$$q(\theta) = v - 3\theta + 2\theta = q_{FB}(\theta) - 2(\theta - \theta).$$

(12)

This output $q(\theta)$ is no longer efficient but still fully revealing and is uniquely determined with the equilibrium range $Q = [q(\theta), q(\theta)]$. In contrast, the equilibrium contributions which support this output are not uniquely determined. The intuition for this fact is similar to the case of the risk-neutral agent: there are many possible ways to redistribute the principals’ payoffs. To see this in the simplest way note that we may determine the payoffs for the principal $P_i$ using the value of the reservation payoff $U_{-i}(\theta)$ from (10). Because $q_{-i}(\theta) < q(\theta)$ the reservation payoff depends on how $t_{-i}(q)$ is extended beyond the equilibrium range $Q$. As there are many ways to extend the contribution schedules, we have a multiplicity of equilibria in the model.

To reduce the number of equilibria we, following Martimort and Stole (2009a), define the natural equilibrium as the contribution schedules defined by the first-order conditions (11) and the equilibrium output (12) not only for the equilibrium range $Q$ but also for the range out-of-equilibrium as long as they remain non-negative.

Definition 2. A natural contribution schedule of the principal $P_i$ is defined for all $q$ by (11).

Using the equilibrium output (12) to obtain $\theta(q)$ and taking the integral in (11) lead to

$$t_{i}(q) = \max \left\{ \frac{2v_1 - v_2 + \theta}{3} q - \frac{q^2}{12} - C_i, 0 \right\},$$

(13)

where $C_i$ is a constant. Martimort and Stole (2009a) provide justification for such a refinement as these natural contributions are robust to perturbations of the support of the parameter $\theta$. Indeed, if we increase $\theta$, keeping $\hat{\theta}$ unchanged, the expression (13) remains the same. We have the following

Proposition 3. In the common agency game with ex-post contracting:

1. The equilibrium output is

$$q(\theta) = v - 3\theta + 2\theta.$$

2. The equilibrium contributions are given by (13) with constants $C_i$ determined by $t_{i}(\bar{\theta}) = 0$.

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6 Non-negativity is imposed only for $\sigma = \infty$ case.
Note that the optimal outputs for polar cases of risk neutrality and infinite risk-aversion determine the range of equilibrium outputs for intermediate risk-aversion. The outputs in these two polar cases are separating for all $\theta$. More precisely, they are strictly decreasing over the whole range of $\theta$.

5. Common agency model: general case

In this section, we consider a general common agency model with the risk-averse agent. In the literature, two cases of common agency modeling considered are intrinsic and delegated common agency games (see Martimort and Stole, 2009b; Calzolari and Scarpa, 2008). In the intrinsic common agency game, the agent upon observing the principals’ offers chooses between joint contracting and non-participation (all-or-nothing participation). In the delegated common agency game, the agent chooses the set of the principals with whom to contract, i.e., the participation decision with any of the principals is delegated to the agent. To have a benchmark for comparison with the one-principal case in section 3, we start with the intrinsic common agency and then we show that under some conditions the equilibrium of this game is also an equilibrium of the delegated common agency game.

In the intrinsic common agency game, the best-response program $(q_i)$ of the principal $P_i$ to the contribution schedule $t_{-i}(q)$ offered by $P_{-i}$ can be written as

$$\text{Max}_{q_i} \int_{\theta} \left[ q_i(\theta) - \frac{q_i^2(\theta)}{4} - \theta q_i(\theta) - U(\theta) + t_{-i}(\theta) \right] \frac{d\theta}{\Delta \theta},$$

subject to (5) and (6).

To solve this maximization problem, we consider the optimal control problem

$$\text{Max}_{q, U, z, \lambda} \int_{\theta} \left[ q(\theta) - \frac{q^2(\theta)}{4} - \theta q(\theta) - U(\theta) + t_{-i}(\theta) \right] \frac{d\theta}{\Delta \theta},$$

with state variables $U(\theta), z(\theta), q(\theta)$ and corresponding co-state variables $\lambda_i(\theta), \mu_i(\theta), v_i(\theta)$. The Hamiltonian for this optimal control problem is

$$H_i = \frac{1}{\Delta \theta} \left[ q - \frac{q^2}{4} - \theta q - U + t_{-i}(q) \right] - \lambda_i q - \mu_i \frac{e^{-\theta U}}{\Delta \theta} + v_i c.$$

To characterize the solution of the optimal control problem, we apply the sufficient conditions for optimality (see Arrow theorem in Seierstad and Sydsaeter, 1987, p. 186):

$$U(\theta) = -q(\theta), (\lambda_i(\theta));$$

$$\dot{z}(\theta) = \frac{1 - e^{-\theta U(\theta)}}{\Delta \theta}, (\mu_i(\theta));$$

$$\dot{q}(\theta) = c(\theta), (v_i(\theta));$$

$$v_i(\theta)c(\theta) = 0, c(\theta) \leq 0, v_i(\theta) \geq 0;$$

$$\lambda_i(\theta) = -\frac{\partial H_i}{\partial \theta} = \frac{1}{\Delta \theta} - \frac{\mu_i \sigma}{\Delta \theta} e^{-\theta U(\theta)};$$

$$v_i(\theta) = -\frac{\partial H_i}{\partial q} = -\frac{1}{\Delta \theta} \left( v_i - \frac{q(\theta)}{2} - \theta + t_{-i}(q(\theta)) \right) + \lambda_i(\theta);$$

$$\lambda_i(\theta) = \lambda_i(\theta) = 0, z(\theta) = 0, z(\theta) \geq 0;$$

$$v_i(\theta) = v_i(\theta) = 0, \mu_i \geq 0, \mu_i z(\theta) = 0.$$

In this problem, co-state variables are continuous piece-wise differentiable. If $c(\theta) < 0$ on the non-degenerate interval then we have $v_i \equiv 0$ on this interval and

$$v_i(\theta) = -\frac{\partial H_i}{\partial q} = -\frac{1}{\Delta \theta} \left( v_i - \frac{q}{2} - \theta + t_{-i}(q) \right) + \lambda_i \equiv 0.$$

---

7 The state variable $z(\theta) = \int_{\theta}^\gamma (1 - e^{-\theta U(\theta)}) d\theta/\Delta \theta$ corresponds to the participation constraint $E_q V(U(\theta)) = \int_{\theta}^\gamma V(U(\theta)) d\theta/\Delta \theta \geq 0$. Then the corresponding co-state variable $\mu_i(\theta)$ is a constant.

8 In our model continuity of co-state variables is possible since the principals’ interests are aligned. With the competing principals, Martimort and Semenov (2008) show that co-state variables have to be discontinuous in equilibrium.
Adding up these conditions and using the first-order condition \( t'_1(q(\theta)) + t'_2(q(\theta)) = \theta \), lead to
\[
q(\theta) = v - \theta - \Delta(\lambda_1(\theta) + \lambda_2(\theta)).
\] (21)
Note that, since \( \lambda_i(\theta) > 0 \) on \((\bar{\theta}, \hat{\theta})\), the common agency’s output is always suboptimal inside the support. Differentiating the expression (21) and using (18), yield
\[
\dot{q}(\theta) + 3 = \sigma(\mu_1 + \mu_2)e^{-\sigma v(\theta)}.
\]
Differentiating this expression once more and using the incentive compatibility condition (14) lead to the differential equation:
\[
\ddot{q}(\theta) = \sigma v(\theta)(\dot{q}(\theta) + 3).
\] (22)
This differential equation determines the equilibrium output in the region where it is strictly decreasing. In general the structure of the optimal contract is described in the next proposition.

**Proposition 4.** In the intrinsic common agency game, there exists \( \hat{\sigma}_{CA} \geq 0 \) with the following properties:

1. If \( \sigma \leq \hat{\sigma}_{CA} \), then the equilibrium output \( q(\theta) \) is the unique solution of the differential equation

\[
\dot{q}(\theta) = \sigma v(\theta)(\dot{q}(\theta) + 3),
\]
\[
q(\theta) = q^{MB}(\theta), \quad q(\hat{\theta}) = \hat{q}_{CA}(\sigma), \quad \text{and} \quad 0 > q(\theta) > -3.
\]

2. If \( \sigma > \hat{\sigma}_{CA} \), then in equilibrium, there exist constants \( \bar{\sigma}_{CA}(\sigma) \) and \( \hat{q}_{CA}(\sigma) \) such that

(i) for \( \theta \in [\bar{\theta}_{CA}(\sigma), \hat{\theta}] \), the output \( q(\theta) = \hat{q}_{CA}(\sigma) \),
(ii) for \( \theta \in [(\bar{\theta}, \bar{\theta}_{CA}(\sigma)) \), the output \( q(\theta) \) is the unique solution of the differential equation

\[
\dot{q}(\theta) = \sigma v(\theta)(\dot{q}(\theta) + 3),
\]
\[
q(\theta) = q^{MB}(\theta), \quad q(\hat{\theta}_{CA}(\sigma)) = \hat{q}_{CA}(\sigma), \quad \text{and} \quad 0 > q(\theta) > -3.
\]

**Proof.** See **Appendix A**. \( \square \)

Let \( q^{\infty}_M(\theta) \) and \( q^{\infty}_{CA}(\theta) \) denote the equilibrium outputs for the one-principal and the common agency cases, respectively when the agent is infinitely risk-averse. The equilibrium outputs for the common agency \( q_{CA}(\theta) \) and one-principal case \( q_M(\theta) \) when \( \sigma > \hat{\sigma}_{CA} \) are depicted in the Fig. 1.\(^{10} \) These outputs are strictly decreasing on \([\bar{\theta}, \hat{\theta}_{CA}(\sigma)]\) and are equal to the constant \( \hat{q}(\sigma) \) on \([\bar{\theta}(\sigma), \hat{\theta}]\) for \( j \in \{M, CA\} \).

The solution for the case of the common agency resembles the solution for the one-principal case. However, the area of bunching is larger under common agency than in merged principal case. Intuitively the contractual externality between principals exacerbates their inability to screen the agent’s preferences in a larger region compare to the case when this externality is internalized. We establish the locations of the optimal outputs for these two cases for small \( \sigma \). These outputs are the solutions of Eqs. (8) and (22) with boundary conditions \( \dot{q}(\theta) = v - \theta \) and \( q(\hat{\theta}) = v - \hat{\theta} \). Comparing these solutions leads to

**Corollary 1.** If \( \sigma \leq \hat{\sigma}_{CA} \), then the output produced by the common agent is always lower than the output produced for the merged principal:
\[
q_{CA}(\theta) < q_M(\theta), \quad \forall \theta \in [\theta, \hat{\theta}].
\]

**Proof.** See **Appendix A**. \( \square \)

\(^9 \) \( \lambda_i(\theta) \) is strictly concave on \((\bar{\theta}, \hat{\theta})\) and using the transversality condition \( (20) \) leads to \( \lambda_i(\theta) > 0 \) on \((\bar{\theta}, \hat{\theta})\).
\(^{10} \) The figure is constructed using numerical simulations. In this case, the level of output \( q_M(\sigma) \) in the bunching zone (for any particular value of \( \sigma \)) is greater than \( q_{CA}(\sigma) \).
If the solution of Eqs. (8) or (22) with boundary conditions \( q(\bar{\theta}) = \nu - \bar{\theta} \) and \( q(\tilde{\theta}) = \nu - \tilde{\theta} \) is strictly decreasing then it represents the equilibrium output for the corresponding game. Corollary 1 and convexity of equilibrium outputs lead to \( q_{\text{CA}}(\theta) > q_{\text{M}}(\theta) \), for all \( \theta \). Therefore, if the solution of (8) is increasing at \( \tilde{\theta} \); \( q_{\text{M}}(\tilde{\theta}) > 0 \) then \( q_{\text{CA}}(\tilde{\theta}) > 0 \). From this, we conclude that, if for a particular \( \sigma \leq \tilde{\sigma}_{\text{CA}} \), the equilibrium output for the one-principal case is separating then the equilibrium output for common agency case is also separating. Together with monotonicity of the output in \( \sigma \), this leads to

\[ \tilde{\sigma}_{\text{CA}} < \tilde{\sigma}_{\text{M}}. \]

**Delegated common agency game:** the assumption in the intrinsic common agency game, that the agent cannot refuse some offers is not always easy to justify (see, for example, Calzolari and Scarpa, 2008). The delegated common agency game, when the agent may contract with each principal individually leads to a more constrained problem for determining the best-response: for each principal \( P_i \), we add to the problem \((\mathcal{P}_i)\) the full participation constraint (7) which says that the agent accepts both contracts in equilibrium. Let \( p_i \) denote the Lagrange multiplier corresponding to this constraint. The best-response problem for \( P_i \) is determined by the constraints (14)–(20) together with the constraints:

\[
\begin{align*}
    &p_i(E_i V(U(\theta)) - E_{i-1} V(U_i(\theta))) = 0; \\
    &p_i \geq 0, E_i V(U(\theta)) \geq E_{i-1} V(U_i(\theta)).
\end{align*}
\]  

(24)

We prove that under some conditions, the equilibrium for the intrinsic common agency game found in Proposition 4 is also the equilibrium of the delegated common agency game. To satisfy this, the principals must be sufficiently close to each other in valuations \( \nu_i \). Otherwise, the principal whose values the good more has an advantage in the delegated common agency and she can offer the contract which may lead to exclusive dealing. Secondly, the principals should have sufficiently high joint valuation \( \nu \).

Note that, in the intrinsic common agency game it is not important how contributions are extended out of the equilibrium range \( Q \) while in the delegated common agency game, these extensions are important since the way they are extended affects the full participation constraints (24). Therefore, we consider the natural equilibrium contribution schedules which are given by (23) everywhere.\(^{12}\) In this case we have the following:

**Proposition 5.** Assume that \( |v_1 - v_2| \leq \tilde{\theta} \) and \( \nu \geq \bar{\nu} \). Then the equilibrium of the intrinsic common agency game in Proposition 4 for \( \sigma \leq \tilde{\sigma}_{\text{CA}} \) is also a natural equilibrium of the delegated common agency game.

**Proof.** See Appendix A. \( \square \)

---

\(^{11}\) See (e) in proof of Proposition 4.

\(^{12}\) Transfers (23) are obtained from the first-order conditions for the Hamiltonians \( H_i \). We may rewrite them as \( t_i(q) = v_i q - q^2/4 - \Delta(\nu) - \int_0^{\theta(q)} \lambda_i(\theta)q(\theta) d\theta - C_i \), where \( \theta(q) \) is the inverse of the equilibrium output.
The intuition for this result is the following: when the joint valuation of the congruent principals is high, they offer non-negative everywhere contribution schedules. Then it is a weakly dominant strategy for the agent to accept both offers. Thus, the full participation constraints (24) are not binding.

Comparative statics with respect to \( \sigma \): Martimort and Stole (2009a) show that in the case of the risk-neutral agent and diminishing support of distribution of \( \theta \), the contribution schedules of natural equilibria always keep track of contractual externality and do not converge to truthful contributions. Our goal is to show that this is not true in the case of the risk-averse agent and diminishing coefficient of risk-aversion \( \sigma \).

Propositions 4 and 5 determine the solution of the common agency problem for low levels of risk-aversion. We have

**Proposition 6.** The set of equilibrium payoffs achieved in limit as \( \sigma \to 0 \) is a subset of equilibrium payoffs achieved with truthful equilibria.

**Proof.** See Appendix A. \( \square \)

Proposition 6 states that, if the coefficient of risk-aversion \( \sigma \) goes to zero, the contributions in the limit are truthful. The intuition is the following: assume that the support of \( \theta \) is finite and any state occurs with a non-zero probability. Since the contracting takes place ex-ante, the truthfulness of the contributions has to be satisfied for all values of \( \theta \) in the support. This leads to the contributions which are truthful everywhere. Proposition 6 shows that this intuition applies even when each state occurs with zero probability. However, even a small risk-aversion distorts the truthfulness of the contributions. This is reflected in the term \( \Delta \theta \int_0^\theta \lambda_i(\theta)\tilde{q}(\theta)\,d\theta \) in Eq. (23).

**Appendix A.**

**Proof of Proposition 2.** The joint surplus \( W_S(\theta) = \max_q(\sum_{i \in S}(q - \theta q)) \) of the coalition \( S \in \{1, 2, (1, 2)\} \) of the principals and the agent at state \( \theta \) is given by

\[
W_i(\theta) = (v_i - \theta)^2, \quad \text{for} \quad i = 1, 2, \quad \text{and} \quad W_{12}(\theta) = \frac{(v - \theta)^2}{2}.
\]

Properties of the cooperative game are determined by the sign of the difference \( W_1 + W_2 - W_{12} \), where \( W_S = \int_0^\theta W_S(\theta)\,d\theta/\Delta \theta \) is the characteristic function of this game. This difference is equal to

\[
\int_\theta^\beta \left((v_1 - v_2)^2 + 3\theta^2 - 2v\theta\right)\,d\theta.
\]

Proposition 2 then follows. \( \square \)

**Proof of Proposition 4.** Firstly, we obtain the expression (23) for contributions. Conditions (19) imply that at the points of differentiability of contributions and output the following holds

\[
\dot{t}_i(\theta) = -v_iq(\theta) + \frac{q(\theta)\dot{q}(\theta)}{2} + \theta \dot{q}(\theta) + \Delta \theta \lambda_i(\theta)\tilde{q}(\theta).
\]

Using the first-order condition for maximization \( \dot{t}_1(\theta) + \dot{t}_2(\theta) = \theta \dot{q}(\theta) \), we obtain

\[
t_i(\theta) = v_iq(\theta) - \frac{q^2(\theta)}{4} - \Delta \theta \int_0^\theta \lambda_i(\theta)\tilde{q}(\theta)\,d\theta - C_i.
\]

To obtain the equilibrium output we adopt the original proof for the one-principal case in Salanié (1990). From (20), we have \( 0 = \lambda_i(\theta) - \lambda_i(\theta) = 1 - \sigma \mu_i \int_0^\theta (1/\Delta \theta)e^{-\sigma U(\theta)}\,d\theta \). Therefore, \( \mu_i > 0 \) and from (20), we have \( \ddot{x}(\theta) = 0 \). Thus, \( \int_0^\beta (1/(\Delta \theta)e^{-\sigma U(\theta)})\,d\theta = 1 \), and from this, we conclude that \( \mu_1 = 1/\sigma \).

(a) Denote by \( \lambda(\theta) \) the sum \( \lambda_1(\theta) + \lambda_2(\theta) \) and by \( \psi(\theta) \) the sum \( v_1(\theta) + v_2(\theta) \). Note first that the optimal output may not have interval of bunching and then separation. Suppose to the contrary that there exist \( \theta_1, \theta_2, \theta_3 \) such that \( \psi(\theta) \) is constant on \([\theta_1, \theta_2] \) and strictly decreasing on \([\theta_2, \theta_3] \). Using the first-order condition for the agent’s maximization (3), we obtain from (19) \( \ddot{v}(\theta) = -(v - q(\theta) - \theta)/\Delta \theta + \lambda(\theta) \). From (17), we obtain that if \( c(\theta) < 0 \) in the non-degenerate interval then \( \psi(\theta) \equiv 0 \) in this interval. Then the co-state equations lead to \( \dot{\lambda}(\theta) = 2/\Delta \theta - (2/\Delta \theta)e^{-\sigma U(\theta)} \) and \( \ddot{v}(\theta) = \dot{q}(\theta) + 3/\Delta \theta - (2/\Delta \theta)e^{-\sigma U(\theta)} \).

Consider continuous and piece-wise differentiable function \( \dot{v}(\theta) \) given by \( \dot{v}(\theta) = -(v - q(\theta) - \theta)/\Delta \theta + \lambda(\theta) \). Since \( c(\theta) < 0 \) on \([\theta_2, \theta_3] \), \( \ddot{v}(\theta) = 0 \) in this interval. Therefore, \( \ddot{v}(\theta) = (\dot{q}(\theta) + 3)/\Delta \theta - (2/\Delta \theta)e^{-\sigma U(\theta)} = 0, \forall \theta \in (\theta_2, \theta_3) \). By continuity of \( U(\theta) \), we obtain \( \ddot{v}(\theta) = 0 < 3/\Delta \theta - (2/\Delta \theta)e^{-\sigma U(\theta)} = \ddot{v}(\theta_2) \). Since \( \ddot{v}(\theta) = (2\sigma/\Delta \theta)U(\theta)e^{-\sigma U(\theta)} < 0 \) on \([\theta_1, \theta_2] \), we have \( \ddot{v}(\theta_1) > 0 \). Continuity of \( \dot{v}(\theta) \) implies that \( q(\theta) \) cannot be strictly decreasing on the interval to the left of \( \theta_1 \). This yields
that \( q(\theta) \) is constant on the interval \([\bar{\theta}, \tilde{\theta}]\) and \( v(\theta) > 0 \) on \((\bar{\theta}, \tilde{\theta})\). Since \( v(\theta) = 0 \) on \([\tilde{\theta}, \theta_2]\) we have \( v(\theta) < 0 \) on \((\bar{\theta}, \tilde{\theta})\). But \( v(\theta) \) is convex on \([\bar{\theta}, \tilde{\theta}]\) and if \( v(\bar{\theta}) = v(\tilde{\theta}) = 0 \), then it implies that \( v(\theta) \) must be positive on some interval belonging to \([\bar{\theta}, \tilde{\theta}]\). Contradiction. Note that since \( c > 0 \) on \([\bar{\theta}, \tilde{\theta}]\) for some \( \bar{\theta} \) and \( v(\theta) = 0 \), we have \( q(\theta) = v - \bar{\theta} \). Therefore, there is no distortion at \( \bar{\theta} \).

(b) From (a) there exist \( \tilde{\theta} \in (\bar{\theta} + 2\theta/3, \tilde{\theta}) \) and \( \tilde{q} \in (v - 3\tilde{\theta} + 2\theta, v - \bar{\theta}) \) such that the equilibrium output in the interval \([\tilde{\theta}, \tilde{\theta}]\) is the solution of the following differential equation:

\[
\begin{align*}
\bar{q}(\theta) &= \sigma q(\theta)(q(\theta) + 3); \\
\bar{q}(\tilde{\theta}) &= v - \tilde{\theta}, \quad \tilde{q}(\tilde{\theta}) = \bar{q}, \quad -3 < \tilde{q}(\theta) < 0. 
\end{align*}
\]

The existence and uniqueness of the solution of this system was established in Salanié (1990). We briefly sketch the proof. With the Eq. (25), we associate the following equation:

\[
\begin{align*}
\bar{q}(\theta) &= \sigma q(\theta)(q(\theta) + 3); \\
\bar{q}(\tilde{\theta}) &= \bar{q}, \\
\tilde{q}(\tilde{\theta}) &= \bar{q}, \\
\tilde{q}(\theta) &= s,
\end{align*}
\]

where \( s > -3 \) is a parameter. This equation, by theorem of Cauchy-Lipschitz, has a unique, parameterized by \( s \), solution which we denote by \( \bar{q}(\theta, s) \). The function \( q(\theta, s) \) is continuous and strictly increasing in \( s \) (Lemma 3, Salanié, 1990). It can be shown that \( v - \bar{\theta} \) belongs to the image of the function \( q(\theta, s), s > -3 \) (Lemma 4, Salanié, 1990). Therefore, there exists a unique \( s^* > -3 \) such that \( \bar{q}(\theta, s^*) = v - \bar{\theta} \). We set \( \tilde{q}(\theta) = q(\theta, s^*) \) for all \( \theta \in [\bar{\theta}, \tilde{\theta}] \).

(c) Consider now the set of solutions of (25) with \( \theta = \tilde{\theta} \) and \( q(\theta) = v - \bar{\theta} \). The solution from this set will be the equilibrium output if and only if it is strictly decreasing for all \( \theta \). Our goal is to show that indeed perfect separation is possible for some values of the coefficient of risk-aversion. We denote by \( \bar{q}(\theta, \sigma) \) the solution of (25) with \( \tilde{\theta} = \tilde{\theta} \) and \( \tilde{q}(\theta) = v - \bar{\theta} \). We prove that, for any fixed \( \theta \in [\bar{\theta}, \tilde{\theta}] \), the function \( q(\theta, \sigma) \) is decreasing in \( \sigma : \partial q(\theta, \sigma) / \partial \sigma < 0 \). Since \( q(\tilde{\theta}, \sigma) = v - \bar{\theta} \) and \( q(\bar{\theta}, \sigma) = v - \bar{\theta} \), we have \( \partial q(\tilde{\theta}, \sigma) / \partial \sigma = \partial q(\bar{\theta}, \sigma) / \partial \sigma = 0 \). Suppose to the contrary that there is a pair \( (\tilde{\theta}, \tilde{\sigma}) \) such that \( \partial q(\tilde{\theta}, \tilde{\sigma}) / \partial \sigma > 0 \). Then for the function \( \psi(\theta) = q(\theta, \tilde{\sigma}) / \partial \sigma \) there exists a local interior maximum \( \hat{\theta} \) such that \( \psi(\hat{\theta}) > 0 \), \( \psi'(\hat{\theta}) = 0 \), and \( \psi''(\hat{\theta}) \leq 0 \). Differentiating the equation \( \partial^2 q(\theta, \tilde{\sigma}) / \partial \sigma^2 = \bar{q}(\tilde{\theta}, \tilde{\sigma}) \partial q(\theta, \tilde{\sigma}) / \partial \sigma + 3 \) with respect to \( \sigma \) and evaluating the result at \((\hat{\theta}, \tilde{\sigma})\) we have

\[
0 \geq \frac{\partial^3 q(\hat{\theta}, \tilde{\sigma})}{\partial \sigma^3} = q(\hat{\theta}, \tilde{\sigma}) \left[ \frac{\partial q(\hat{\theta}, \tilde{\sigma})}{\partial \sigma} + 3 \right] + \bar{q}(\tilde{\theta}, \tilde{\sigma}) \frac{\partial \bar{q}(\tilde{\theta}, \tilde{\sigma})}{\partial \sigma} \partial q(\hat{\theta}, \tilde{\sigma}) / \partial \sigma + 3 + \bar{q}(\tilde{\theta}, \tilde{\sigma}) \frac{\partial^2 q(\hat{\theta}, \tilde{\sigma})}{\partial \sigma^2}. 
\]

The third term on the RHS is zero, the first and the second ones are positive. Therefore contradiction.

(d) The set of separating outcomes defined in (c) does not exhaust equilibrium outcomes for all coefficients of risk-aversion. We show that there exists \( \sigma \) such that the solution of (25) is not strictly decreasing at \( \tilde{\theta} \): \( \bar{q}(\tilde{\theta}) > 0 \). If we assume that \( \bar{q}(\tilde{\theta}) \leq 0 \) then \( \bar{q}(\tilde{\theta}) \leq 0 \) for all \( \theta \in [\bar{\theta}, \tilde{\theta}] \). Therefore, we have

\[
\bar{q}(\tilde{\theta}) - \bar{q}(\tilde{\theta}) = \int_{\bar{\theta}}^{\tilde{\theta}} \bar{q}(\theta) \, d\theta = \sigma \int_{\bar{\theta}}^{\tilde{\theta}} q(\theta)(q(\theta) + 3) \, d\theta \\
= 3\sigma \int_{\bar{\theta}}^{\tilde{\theta}} q(\theta) \, d\theta + \sigma \frac{q^2(\theta)(v - \tilde{\theta} + \Delta \theta)}{2} \geq 3\sigma \Delta \theta(v - \tilde{\theta} + \Delta \theta/6) + \sigma \frac{\Delta \theta}{2}(2v - 2v - \frac{\Delta \theta}{2}) = \sigma \Delta \theta(4v - 3\tilde{\theta} - \tilde{\theta}) > 0.
\]

The last inequality follows from the assumption \( v \geq \tilde{\theta} \). Therefore, \( \bar{q}(\tilde{\theta}) - \bar{q}(\tilde{\theta}) \geq \sigma M > 0 \) which contradicts the conditions \(-3 < \bar{q}(\theta) < 0\) when \( \sigma \) is sufficiently high. \(\Box\)

**Proof of Proposition 5.** To prove the proposition we assume that \( p_i = 0 \) for \( i = 1, 2 \). Then for the equilibrium in Proposition 4 to be the equilibrium of the delegated common agency game we need to check constraints (24). We rewrite these constraints as

\[
\int_{\bar{\theta}}^{\tilde{\theta}} e^{-\sigma U_1(\theta)} \, d\theta \geq \int_{\bar{\theta}}^{\tilde{\theta}} e^{-\sigma U_2(\theta)} \, d\theta, \quad i = 1, 2.
\]

(a) Firstly, we prove that contributions \( t_i(\theta) \) are strictly decreasing. Differentiating (23) with respect to \( \theta \) yields

\[
t_i(\theta) = \frac{q(\theta)}{2} - \Delta \theta \lambda_i(\theta).
\]
By (4), we have $\dot{q}(\theta) < 0$. The derivative of $q(\theta)/2 + \Delta \theta \lambda_i(\theta)$ is equal to $(2e^{-\sigma U(\theta)} - 3)/2 + 1 - e^{-\sigma U(\theta)} = -(1/2) < 0$. Hence, if $v_1 - q(\theta)/2 - \Delta \theta \lambda_i(\theta) \geq 0$, the expression $v_1 - q(\theta)/2 - \Delta \theta \lambda_i(\theta)$ is positive and then the contributions $t_i(\theta)$ are decreasing. The conditions $v_1 - q(\theta)/2 - \Delta \theta \lambda_i(\theta) \geq 0$ are satisfied if $|v_1 - v_2| \leq \bar{\theta}$. Therefore, the contributions are decreasing in $\theta$.

(b) The outputs $q(\theta)$ and $q_i(\theta)$ are defined by equations $t'(q(\theta)) = \theta$ and $t'(q_i(\theta)) = \theta$, respectively. From (27) we obtain

$$t'(q(\theta)) = v_1 - q(\theta) - \Delta \theta \lambda_i(\theta).$$

For the natural equilibrium this expression for contributions is valid for all $q$ with $\theta = \theta(q)$ in the equilibrium range and $\theta$ equals to constant out of equilibrium range.$^{13}$ This leads to

$$q_i = 2(v_1 - \theta - \Delta \theta \lambda_i) \quad \text{and} \quad q = v - \theta - \Delta \theta \lambda.$$

Therefore, we have

$$q_i = q + (v_1 - v_i) - \theta - \Delta \theta (\lambda_i - \lambda_i). \quad (28)$$

(c) Using (28), we can reformulate the full-participation constraint (26) as

$$\int_{\mathcal{D}} e^{-\sigma U(\theta)} d\theta = \int_{\mathcal{D}} e^{-\sigma U(\theta) + \sigma(t_i(\theta) + (v_1 - v_i)\theta - \theta^2 - \Delta \theta \lambda_i - \lambda_i)} d\theta \geq \int_{\mathcal{D}} e^{-\sigma U(\theta)} d\theta,$$

which leads to a sufficient condition

$$t_i(\theta) + (v_1 - v_i)\theta - \theta^2 - \Delta \theta \lambda_i - \lambda_i \theta \geq 0. \quad (29)$$

Since the function on the LHS of (29) is decreasing it is sufficient to prove this inequality for $\hat{\theta}$. Using $|v_1 - v_2| \leq \bar{\theta}$, the LHS is equal to

$$t_i(\bar{\theta}) + (v_1 - v_i)\bar{\theta} - \bar{\theta}^2 \geq (v - \bar{\theta}) \frac{v + \bar{\theta}}{4} - 2\bar{\theta} - \bar{\theta}^2.$$  

We assume $v \geq 3\bar{\theta}$. Then $(v - \bar{\theta}) \frac{v + \bar{\theta}}{4} - 2\bar{\theta} - \bar{\theta}^2 \geq 2\bar{\theta}^2 - 2\bar{\theta} - \bar{\theta}^2 \geq 0$. Thus, the full-participation constraint is satisfied. □

**Proof of Corollary 6.** Fix $\sigma$ and consider the parameterized by $\alpha$ equation

$$\frac{\partial^2 q(\theta, \alpha)}{\partial \alpha^2} = \sigma q(\theta, \alpha) \left[ \frac{\partial q(\theta, \alpha)}{\partial \theta} + \alpha \right], \quad (30)$$

where the parameter $\alpha = 2$ in the one-principal case and $\alpha = 3$ in the common agency case. Our purpose is to prove that $\partial q(\theta, \alpha)/\partial \alpha \leq 0$. From the border conditions, we have $\partial q(\bar{\theta}, \alpha)/\partial \alpha = 0$. Suppose that there exists $(\bar{\theta}, \bar{\alpha})$ such that $\partial q(\bar{\theta}, \bar{\alpha})/\partial \alpha > 0$. Then there exists a local maxima of $\partial q(\theta, \bar{\alpha})/\partial \alpha$ at point $(\tilde{\theta}_{\max}, \bar{\alpha})$ with $\partial^2 q(\tilde{\theta}_{\max}, \bar{\alpha})/\partial \theta \partial \alpha = 0$ and $\partial q(\tilde{\theta}_{\max}, \bar{\alpha})/\partial \alpha > 0$. Then Eq. (30) yields

$$0 \geq \frac{\partial^2 q(\tilde{\theta}_{\max}, \bar{\alpha})}{\partial \theta^2 \partial \alpha} = \sigma \frac{\partial q(\tilde{\theta}_{\max}, \bar{\alpha})}{\partial \alpha} \left[ \frac{\partial q(\tilde{\theta}_{\max}, \bar{\alpha})}{\partial \theta} + \alpha \right] + \sigma q(\tilde{\theta}_{\max}, \bar{\alpha}) > 0.$$

Hence contradiction. Therefore $\frac{\partial q(\theta, \alpha)}{\partial \alpha} \leq 0$ and from this $q_{CA}(\theta) < q_{M}(\theta)$, $\forall \theta \in (\bar{\theta}, \bar{\theta})$. □

**Proof of Proposition 1.** Let us denote the output and the contribution for a particular coefficient of risk-aversion $\sigma \leq \sigma_{CA}$ as $q(\theta, \sigma)$ and $t_i(\theta, \sigma)$, respectively. The equilibrium contribution schedule $t_i(\theta, \sigma)$ for the principal $P_i$ is determined by

$$t_i(\theta, \sigma) = v_i q(\theta, \sigma) - \frac{q^2(\theta, \sigma)}{4} - \Delta \theta \int_\bar{\theta}^{\theta} \lambda_i(\theta, \sigma) q(\theta, \sigma) d\theta - C_i. \quad (31)$$

$^{13}$ These constants are $\hat{\theta}$ for $q < q(\bar{\theta})$ and $\bar{\theta}$ for $q > q(\bar{\theta})$. 
Consider the function \( \int_{\theta}^{0} \lambda_{i}(\theta, \sigma) \dot{q}(\theta, \sigma) d\theta \). Co-state \( \lambda_{i}(\theta, \sigma) = (1/\Delta)(\int_{\theta}^{0} (1 - e^{\alpha U(\theta)}) d\theta \) converges to zero when \( \sigma \to 0 \) by continuity of \( U(\theta) \) in \( \sigma \). On the other hand \( \dot{q}(\theta) \) is bounded: \( -3 < \dot{q}(\theta) < 0 \). This implies that \( \lim_{\sigma \to 0} \int_{\theta}^{0} \lambda_{i}(\theta, \sigma) \dot{q}(\theta) d\theta = 0 \). From this we conclude that if the coefficient of risk-aversion \( \sigma \) goes to zero, the contributions in the limit are truthful and given by

\[
t_{i}(\theta, 0) = v_{i}q(\theta, 0) - \frac{q^{2}(\theta, 0)}{4} - C_{i}. \]

\[\square\]

References